University of California Transportation Center
UCTC Dissertation No. 160

Essays on Congestion Economics

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University of California, Santa Barbara
2008

UNIVERSITY OF CALIFORNIA

Santa Barbara

Essays on Congestion Economics

A Dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Economics

by

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June 2007

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ACKNOWLEDGEMENTS

I would like to thank Theodore Bergstrom, Gary Charness, Jon Sonstelie, Rod Garratt, Robin Lindsey, participants of the UC Santa Barbara Theory Underground seminar group, and participants of the First International Conference on Funding Transportation Infrastructure for helpful comments.

I am extremely grateful to Perry Shapiro for help on Chapter II, especially for assistance with the sections related to the Transition Probabilities, and the Individual Response to Current State and Stochastic Choice framework.

I am grateful for funding from a Doctoral Dissertation Research Grant from the University of California Transportation Center and from the Department of Economics at the University of California Santa Barbara.

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ABSTRACT

Essays on Congestion Economics

by

John Lawrence Hartman

Traffic congestion is a substantial time cost for many urban commuters. This dissertation first studies the response of subjects in experimental settings in which subjects choose between a short direct route that becomes increasingly congested as more people travel on it and a more indirect route that does not become congested. More specifically, I investigate three different toll implementations. Consistent with previous experiments, my first toll design imposes monetized homogeneous time costs. Within this framework the implementation of a toll comes very close to achieving the efficient use of the travel network predicted by theory. Two other toll designs implement heterogeneity in the subject pool. In the first design, I implement a design that more closely simulates boring commutes by forcing subjects to sit and wait for a period of time after the experiment where the length of time they have to wait is an increasing function of experimental travel time. Paying the toll can reduce waiting times. There is substantial heterogeneity in outcomes between groups, which is likely due to the distribution of values of time by session. This may help

explain why similar traffic networks have different commuting patterns when they serve different populations. The second toll design imposes monetized heterogeneous time costs. As the level of heterogeneity rises, the number of travelers on each route becomes more stable. This contrasts with other experimental work, which shows a substantial level of instability in a homogeneous framework. Finally, I also analyze various models to study behavior in a no-toll homogeneous framework. While no theory explains individual behavior well, I do find that some theories explain aggregate behavior quite well.

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1. Introduction

Traffic congestion in the United States costs the average urban rush hour traveler 47 hours and 28 gallons of gasoline per year, for an annual cost of $794 per traveler.[1] The extra gasoline consumed also produces 546 additional pounds of carbon dioxide,[2] along with other pollutants such as carbon monoxide.[3] Much of this congestion is the result of individuals failing to account for externalities. Traffic volume often exceeds the optimal level because of excessive driving by individuals who fail to internalize the congestion time costs they impose on others. It is for this reason that the use of tolls can improve the efficiency of congested roads and highways.

While the theoretical foundations of congestion externalities have long been understood, only recently has congestion been studied experimentally. In general,

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[1] These numbers are taken from the Texas Transportation Institute's 2005 Urban Mobility Study, http://mobility.tamu.edu/ums/congestion_data/tables/all_85_urban_areas.pdf, which estimates congestion costs from 2003. From Table 2 in this paper (see http://mobility.tamu.edu/ums/congestion_data/tables/national/table_2.pdf), traffic congestion results in more than 3.7 billion hours in travel delay and over 2.2 billion extra gallons of gasoline consumption, for a total congestion cost of over $63 billion annually.

[2] Using official energy statistics from the U.S. government, the Energy Information Administration (see http://www.eia.doe.gov/oiaf/1605/factors.html) estimates that each gallon of gasoline used for driving emits over 19.5 pounds of carbon dioxide.

[3] According to the Environmental Protection Agency (see http://www.epa.gov/air/airtrends/aqtrnd01/carbon.html), about 60 percent of carbon monoxide (CO) emissions in the United States are emitted by motor vehicles. The website also reports that "(h)igh concentrations of CO generally occur in areas with heavy traffic congestion."

experiments study the formation of congestion under various transportation networks. In most of these experiments subjects must choose between two transportation routes, where one is sometimes more congestible than the other. These studies share two important simplifying assumptions that are not particularly realistic. First, they assume homogeneous time costs. While this simplifies the analysis, it seems quite unrealistic in a world with a wide range of earnings power and hence a wide range of time opportunity costs. Secondly, all of these experiments monetize time costs. It is certainly possible that people respond differently when faced with time costs compared to monetary costs. A high wage person might welcome a higher toll on a congested route, while a lower wage person may prefer the longer commute time rather than having to pay the higher monetary cost on the same route. As all existing experimental evidence comes from experiments that only include monetary costs, previous research is silent on the question of how people react when forced to pay with actual time. This paper uses data from two route choice experiments to address the two issues raised above.

In contrast to previous congestion experiments, one of my two experiments includes both money and time costs. In particular, subjects must physically wait after the experimental rounds. This waiting time, in which a subject must sit and do nothing at the end of the experiment, can be reduced by paying a monetary cost. The addition of waiting time produces an experimental design that more closely resembles a real traffic environment. I can also determine the cost per hour to reduce waiting time.

Another relevant issue related to congestion involves heterogeneity. Theory predicts that subjects with the highest values of time will be more willing to pay a monetary cost rather than a time cost, while those with the lowest values of time will be more willing to pay the time cost. Subjects' values of time are allowed to be heterogeneous, with the possibility that many people either have high or low values of time. Theory typically predicts only one or a few equilibria within each subject group, since almost each person has a strict preference of routes in equilibrium. So few equilibria occur because people typically have strict route preferences when the system is in equilibrium. The number of travelers on each route in equilibrium can also differ depending on the distribution of subjects' values of time. These results are in stark contrast to previous experiments, where subject homogeneity leads to a situation in which subjects can travel on either route in equilibrium and thousands of equilibria possible, leading to coordination problems not observed in actual traffic environments. In the homogeneous case, so many equilibria can occur because once equilibrium occurs, each person is indifferent about the route chosen.

These experimental designs produce several important findings. First, the average experimental results come close to the predictions made by theory. Second, tolls can improve the efficiency of a road network, reducing the total travel time of all travelers. This result holds true whether or not time costs are homogeneous. Third, by imposing waiting time, I can test to see if different experimental groups have different distributions of value of time. While monetized time costs lead to the same predicted result in each experimental group, the use of waiting time does not necessarily lead to the same conclusion. Fourth, increased heterogeneity leads to decreased variance of travelers on each route. Finally, there is some evidence that subjects under substantial time constraints are willing to incur monetary costs in order to reduce time costs.

2. Background and Motivation

2.1. Congestion and Tolls

Vickrey (1963, 1969) shows that individuals drive inefficiently large amounts on unpriced highways because they fail to internalize the congestion time they impose on other drivers. He further shows that differential pricing on streets and highways, similar to that observed in the airline and movie industries, can lead to Pareto improvements. In the case of flights and movies, monopoly deadweight loss is reduced through price discrimination. On roads and highways, differential pricing diverts drivers from more congested routes to less congested routes. In the case of an optimal highway pricing mechanism, the marginal benefit of reduced travel time for the last driver equals the marginal external costs due to increasing travel time to the other drivers. Section 3 develops this idea for the experimental design used in this paper. Further, differential pricing on public roads, in the form of tolls, are not actually costs but rather transfers to the government.[4] The only actual costs generated by tolls are the costs associated with collecting the tolls, costs that

[4] Note, however, that tolls appear to be costs to drivers.

have been significantly reduced with transponder technology.[5] As long as the cost of collecting tolls is less than the benefits of decreased travel times, Pareto improvements are possible.

Although Vickrey's ideas are well established in economics, public policy has leaned more towards increased highway capacity than price rationing. This is particularly problematic since such increased capacity often fails to reduce congestion, and in some cases actually worsens it.[6] In a simple case that does not change drivers' outcomes in equilibrium when capacity is increased, congestion remains the same as long as both routes are used. This case, known as the Pigou-Knight-Downs paradox (see Arnott and Small 1994), uses a network with an uncongested route and a congested route.[7] On the uncongested route, travel time is the same regardless of the number of travelers. On the congested route, the travel time increases for all drivers each time another traveler decides to use the route.

Unlike highway expansion, tolling schemes are capable of reducing congestion in the framework of the Pigou-Knight-Downs paradox, and can therefore lead to Pareto improvements. In equilibrium without tolls, everybody's travel time equals the time on the uncongested route. After the implementation of a toll on the

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[5] Transponders are small devices that are attached to vehicles. These transponders can be detected by devices on highways without drivers having to slow down.

[6] See Arnott and Small (1994) for cases that predict worse congestion when road capacity is increased.

[7] The travel network used in this paper produces the Pigou-Knight-Downs paradox, which "is often called 'the fundamental law of traffic congestion,'" according to Arnott and Small (1994, p. 451).

congested route, nobody can be made worse off, because anybody may still travel the uncongested route with the same travel time. Thus, anybody traveling the tolled congested route must be at least as well off as without the toll. Toll revenues are also available to disburse among the population, increase transportation or other funding, or reduce tax burdens.

Arnott, de Palma, and Lindsey (1990, 1993, 1994) refine Vickrey's ideas using a model with a single route with a bottleneck. In a standard bottleneck situation, only a fixed number of vehicles can pass during a given time period. If there is a point in time in which more vehicles attempt to pass than the capacity can handle, a queue develops. Each person has an ideal arrival time, and also incurs costs for early or late arrival. It is generally assumed that vehicles leave the bottleneck in the order in which they enter it. In this model, tolls are capable of decreasing congestion in equilibrium. In this case, an optimal toll changes throughout the day so that the toll equals the equilibrium queuing costs in the no-toll equilibrium. The new equilibrium results in no waiting time for any driver at the bottleneck, which leads to an efficient outcome.

2.2. Congestion Experiments

In recent years, experiments have been used to explore human behavior on various traffic grids. These experiments pay subjects based on their performance in the experiment, with better performance resulting in higher payouts. The experiments listed below use modifications of the Pigou-Knight-Downs paradox, or other congestible scenarios. Before discussing the finding of these experiments, it is important to emphasize that only one of these experiments (Gabuthy, Neveu, and Denant-Boemont (2006)) uses tolls as a mechanism to reduce traffic flow on a particular route, and none allow for the possibility that travel costs are heterogeneous.

Selten *et al* (2007) modify the two-route network described by the Pigou-Knight-Downs paradox by allowing both routes to be congestible. In this experiment, 18 subjects must travel between two points on either the "main" road or the "side" road, where the side road requires more travel time than the main road if the number of subjects traveling on both routes is the same. Subjects then receive a payout in each round as a function of travel time, with higher travel time resulting in a lower payout. These choices are repeated over 200 rounds, with theory predicting an equilibrium in each round of 12 subjects on the main route. On average, subject route choices come very close to the theoretical predictions. However, since the population in this experiment is homogeneous and no tolls are charged, any subject's route choice can be part of an equilibrium as long as 12 subjects travel the main route, since theory only predicts the number of people on each route. With thousands of equilibria possible, any subject could be on either route in equilibrium, leading to substantial fluctuations of the number of travelers on each route from round to round, even in rounds after equilibrium. Although such fluctuations reject the predictions of pure-strategy equilibrium, the mean number of travelers on each route comes close to this equilibrium.

Chmura and Pitz (2004a and 2004b) modify the payoffs to a minority game structure. In this version of the minority game, travelers on the route with the fewest travelers (in each round) win a positive payment and everyone else receives nothing.[8] Although the minority game framework is useful in many economic settings, it may not be the best way to model the payoffs in a transportation network because commuters typically do not "win" or "lose," but rather incur one of many possible commuting costs.

Two results from the Selten *et al* (2007) and Chmura and Pitz (2004a and 2004b) experiments are worth highlighting. First, a person's payout in one round is negatively correlated to the likelihood that same person will switch routes in the following round. This implies that many subjects think that the "other" route will be the better choice after a relatively bad payout. In other words, many people believe that a relatively bad payout follows another relatively bad payout if they remain on the same route from one round to the next. Second, subjects who switch routes frequently over the course of the experiment tend to have worse overall payouts than those who switch less frequently. These results shed some light on how subjects react when faced with a coordination problem (which is described in more detail in the next subsection), and how their reactions affect overall payoffs.

Three other experiments incorporate either departure or arrival time as a choice variable, the first two in an Arnott, de Palma, and Lindsey (1990, 1993)

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[8] An odd number of subjects in each experimental group ensures that the subjects on only one route receive payment in every round.

framework. In this framework, decreasing congestion costs can reduce total travel costs. Without tolls, any Nash equilibrium is typically not optimal, due to congestion. The first, by Schneider and Weimann (2004), shows that subjects tend to make decisions that result in Nash equilibrium, rather than the optimal outcome. Besides tolls, they also suggest that having each subject play more than one vehicle per experimental round as another solution to this inefficiency. The second of these experiments, by Gabuthy, Neveu, and Denant-Boemont (2006), uses two routes and requires subjects to choose both route and departure time. They find some deterrence to traveling a particular route when the higher of two tolls is used, but the implementation of tolls in this experiment do not decrease total travel costs (excluding tolls). The final experiment incorporates arrival time for a congestible service, such as waiting in line at a bank. This paper, by Rapoport *et al* (2004), is not applicable to most cases of automobile congestion, since the experiment in this paper examines a situation with an explicit opening and closing time and that the benefit of the service is constant throughout the day.

2.3. Coordination and Cooperation

In real traffic environments, thousands of drivers try to determine the "best" route to travel on a given day. Drivers often choose a route based on the information they have about traffic speed on their set of route choices on previous days and/or current traffic reports. Further, at any particular moment in time thousands of drivers are simultaneously making the same set of calculations and route choices.

Without the ability to coordinate, it is unlikely that equilibrium is reached on any given day on many congested routes. Furthermore, cooperation that could lead to more efficient outcomes is not seen much (if at all) on congested routes.

This situation is similar to coordination games.[9] Cooper *et al* (1990) describe coordination games as "a class of symmetric, simultaneous move, complete information games" (p. 218). In these games, multiple Nash equilibria exist. However, there may also exist outcomes that are better for all participants but are not Nash equilibria. For example, in a two-player game in Cooper *et al* (1990), they use an example in which neither of the two Nash equilibria is Pareto optimal. The optimal outcome in this case is not an equilibrium, since one of the two subjects could deviate to produce a better individual outcome.

Consider the grid system of the Pigou-Knight-Downs paradox (see Arnott and Small 1994). In any Nash equilibrium, all subjects have the same travel time.[10] However, if the drivers coordinated their efforts they could decide to have fewer drivers than the equilibrium number on the congested route. This would result in a faster travel time on the congested route. The group traveling the congested route could change each day so that each driver could travel the congested route some of the time, resulting in an outcome that Pareto-dominates any Nash equilibrium.

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[9] For an overview of coordination problems, see Ochs (1995).

[10] Another set of Nash equilibria exists. Let the set of equilibria with equal travel times have q drivers on the congested route. The other set of Nash equilibria has $q - 1$ drivers on the congested route. Although the travel time is lower on the congested route in this case, these are also Nash equilibria because if someone on the uncongested route switches to the congested route, the times then become equal, leading to no change in the travel time of the person who switches. This set of equilibria is not focused on as much for simplicity in the analysis.

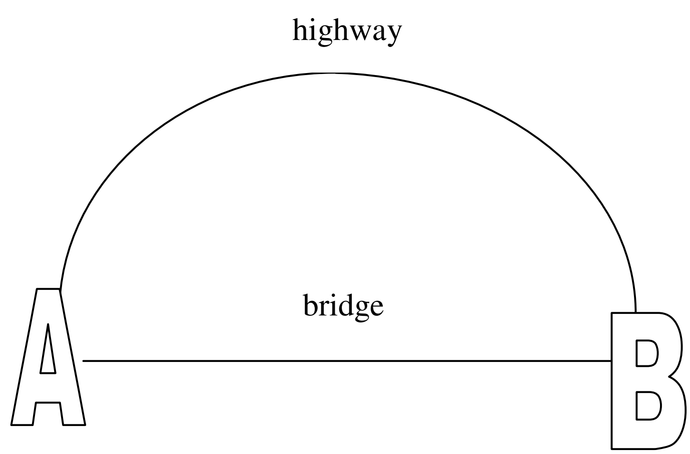
Although all drivers are better off if they all follow the agreement, this outcome is not a Nash equilibrium if they are not bound to follow it. More specifically, it is not a Nash equilibrium because any single driver could deviate and travel the congested route on a day that he is assigned to the uncongested route, leading to a better travel time for that driver. In the next section, I discuss another mechanism to reduce the number of people traveling on congested routes, the use of tolls, which can lead to a Nash equilibrium that results in Pareto improvements.

3. A Two-Route Model With Only One Route That Congests

Suppose a group of people need to travel from point A to point B, and that each person has the option to travel on an uncongested but indirect highway, or a more direct but narrow bridge that gets congested (See Figure 1).[11] In other words, highway travel time is independent of the number of travelers, while travel time on the bridge is an increasing function of bridge traffic. Assume that the per-minute travel time cost is independent of route choice. This rules out the possibility that the more scenic route is preferred, all else being equal. Given this framework, it is easy to show that under the standard assumption of homogeneous travel time costs (using uniform point deductions in an experimental setting) equilibrium occurs when the total cost to commuters (including tolls, if any) is identical on both routes. In

[11] This route grid structure is as in Arnott and Small (1994), and some of the theory is similar to Walters (1961).

Figure 1: A visual of the scenario that subjects see for their travel situation



contrast, under heterogeneous travel time costs, the equilibrium depends on the distribution of the values of time among participants.

3.1. The No-Toll Case Equilibrium Under Homogeneous Time Costs

In the absence of tolls, the only costs of traveling from point A to point B are time costs. In this example, I assume that each person's per-minute travel time costs are homogeneous. For simplicity, I assume that N commuters know the travel time on the bridge (t_B) and the highway (t_H) with certainty. In particular, they know that travel time on the highway is constant and that travel time on the bridge is an increasing linear function of the number of travelers on the bridge (T) with intercept α and slope β, such that:

(1) $t_B = \alpha + \beta T$.

For each additional traveler on the bridge, time increases by β minutes. Based on the above information, drivers can determine the marginal private benefit (MPB) of traveling the bridge relative to the highway. If T drivers travel the bridge, the MPB in minutes of the T^{th} person traveling the bridge is the difference in travel time between the two routes, or $t_H - (\alpha + \beta T)$. To convert the travel time into monetary terms, the time saved needs to be multiplied by the individual's value of time:

(2) $MPB_T = (t_H - (\alpha + \beta T)) \times V_T$,

where V_T represents the value of time for the T^{th} person to travel the bridge. Because

V_T is constant in a homogeneous framework, and time savings decrease linearly with T, MPB is a decreasing linear function, as shown in Figure 2.[12]

I also assume that subjects maximize utility by minimizing travel time. Equilibrium therefore occurs when the travel time on both routes is the same, or at the point where MPB is zero:[13]

$$(3)\ t_B = t_H \quad \Rightarrow \quad \hat{T} = \frac{t_H - \alpha}{\beta}.$$

In all other cases, at least one traveler can increase their utility by switching routes.

Finally, although theory is able to predict the number of people on each route in equilibrium, it is unable to predict the route that any particular person travels. Since any person can travel on either route in equilibrium, any combination of T people on the bridge constitutes an equilibrium. Thus, there exist $\binom{N}{T}$ equilibrium combinations.

3.2. Tolls and Efficiency in a Homogeneous Time Cost Setting

The problem in the no-toll case is that people fail to internalize the additional costs they impose on others when they use the congestible bridge. In a no-toll equilibrium, everyone is just as well off in an environment in which both routes exist

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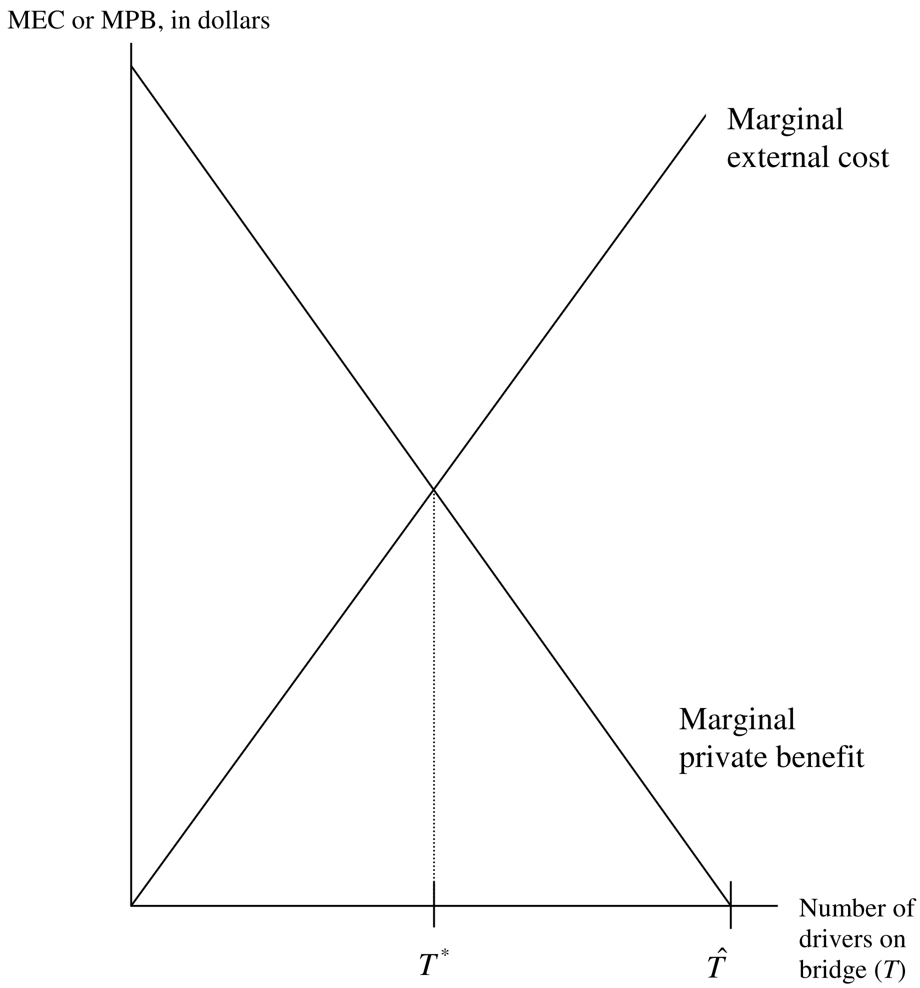
[12] The same cost/benefit analysis in Figure 2 can be done in minutes with the same result, since per minute travel time costs are the same for each person in this case.

[13] In a no-toll scenario, the same equilibrium occurs when drivers have different values of time. The idea of homogeneity of value of time is relevant for the analysis of the toll case, which is described below.

Figure 2: Marginal external cost (MEC) and marginal private benefit (MPB) in a homogeneous value of time case



than in an environment in which only the highway exists.[14] If only the highway exists, adding the bridge adds no social benefit because commuters simply congest the bridge to the point where there is no time gain to traveling the bridge over the highway. Given the negative externalities present on the congested route, a toll on the bridge can effectively optimize its use by reducing the travel time of some of the drivers on this route. At the same time, no toll is needed on the highway because there are no externalities, since congestion is never present by definition.

The optimal toll minimizes drivers' total travel time costs.[15] In a framework with homogeneous values of time, this minimization problem is equivalent to minimizing the total travel time of all drivers, since the monetary equivalent of time costs is the same for all drivers. So if T commuters use the bridge and $(N - T)$ commuters use the highway, then the total travel time for all drivers (TT) is given by:

(4) $TT = Tt_B + (N - T)t_H$.

Minimizing total travel time then gives the optimal distribution of travelers:

(5) $T^* = (t_H - \alpha) / 2\beta$.

Another way of determining T^* is by finding the point where MPB equals the marginal external cost (MEC) on the bridge. MEC is positive because an additional driver on the bridge imposes an additional β minutes to each driver already on the bridge. MEC is then:

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[14] The fact that everyone would be as well off in a no-toll equilibrium with or without the bridge is specific to the Pigou-Knight-Downs paradox. Other paradoxes are presented in Arnott and Small (1994) in which travel times are increased when road capacity increases.

[15] Recall that I assume that tolls are simply transfers to the government. Even though tolls appear to be costs to drivers, they can be rebated back to drivers by the government.

(6) $MEC_T = \sum_{j=1}^{T} (\beta \times V_j)$.

In this case, since both β and V_j and constants in this framework, MEC is an increasing linear function, as seen in Figure 2.

The optimal toll is then defined as the toll that makes travelers indifferent between traveling on the bridge and the highway when T^* drivers travel on the bridge. To find this toll, I must find the monetary equivalent of the difference between highway travel time and optimal bridge travel time. This cost (C) is a linear relation of time, and so I only need to multiply this time difference by the per-minute value of time V:

(7) $C = [t_H - (\alpha + \beta T^*)]V$.

Finally, similar to the previous subsection, there exist $\binom{N}{T^*}$ possible combinations that lead to equilibrium when the optimal toll is imposed.

3.3. Equilibrium with Heterogeneous Values of Time and Tolls

In this situation, I relax the assumption of homogeneous values of time. Once again, I assume that tolls are implemented on the bridge. Thus, no traveler would prefer the bridge if the travel time on this route exceeds that of the highway. As in the previous case with homogeneity, equilibrium always occurs when travel time on the bridge is less than that on the highway. Recall that the MPB of time saved by traveling the bridge is $t_H - (\alpha + \beta T)$ minutes. With a toll of C, the cost savings per minute of travel time if T people travel the bridge is then

$$(8)\ \frac{C_T}{60} = \frac{C}{t_H - (\alpha + \beta T)},$$

with C_T denoting the per-hour toll cost if T drivers travel on the bridge.[16] Equation (8) translates this into an hourly rate of

$$(9)\ C_T = \frac{60C}{t_H - (\alpha + \beta T)}.$$

Without loss of generality, let each person's value of time be V_T per hour such that: $V_1 \geq V_2 \geq \ldots \geq V_N$. If $V_1 \leq C_1$, then nobody travels the bridge in any round. The cost of the toll here is more than the value of time saved by the person with the highest value of time. Generally, each person travels on the route in which the toll paid, if any, plus travel time cost is lower. Under these conditions, people with the highest values of time travel the bridge because they are able to reduce their travel time costs substantially due to their high value of time.[17] Specifically, letting C_T denote the per hour cost in waiting time while T subjects travel the bridge,

- If $C_T < V_T$, the T^{th} person will travel the bridge.
- If $C_T = V_T$, the T^{th} person will be indifferent between the two routes.
- If $C_T > V_T$, the T^{th} person will not travel the bridge.

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[16] Since the per-hour cost is C_T, the per-minute cost is $C_T/60$.

[17] There are sometimes exceptions to this when $V_T \approx V_{T+1}$, which are examined further in the experiment with heterogeneous and monetized time costs.

I will typically assume that the person that is indifferent travels on the bridge, since this is consistent with the Pigou-Knight-Downs paradox. In this case, theory not only predicts the number of people on each route (which is dependent on the distribution of values of time), but also predicts that the subjects that travel the bridge in equilibrium have the highest values of time.

A Discrete Example

A simple example incorporating heterogeneous waiting times requires a distribution of waiting time values along with specifying the right hand side parameters for equation (9). For illustrative purposes, column 2 in Table 1 lists a hypothetical set of values of time for the N = 18 participants in descending order, while the values for C_T are determined from C = \$0.25, t_H = 30, α = 28.4, and β = 0.2. With $\hat{T} = 8$ in this example, C_T is undefined when T = 8, and negative when T > 8. In these uninteresting cases, C_T is listed as N/A in Table 1.

Here, the person with the highest value of time travels the bridge, due to the \$100 per hour value of time being larger than the \$10.71 per hour cost of traveling the bridge. Similarly, the person with the second highest value of time also travels the bridge since \$80 > \$12.50. This continues through the fifth highest value of time, with \$30 > \$25. The sixth highest value of time, with a value of time of \$29 per hour, does not travel the bridge since the additional congestion increases the per-hour cost of traveling the bridge to \$37.50.[18] Thus, five people travel the bridge in equilibrium. Specifically, the five people with the highest values of time travel the bridge.

Table 1: V_T and C_T for Heterogeneous Value of Time Example

T	V_T, in dollars	C_T, in dollars
1	100.00	10.71
2	80.00	12.50
3	70.00	15.00
4	50.00	18.75
5	30.00	25.00
6	29.00	37.50
7	22.00	75.00
8	21.00	N/A
9	20.00	N/A
10	20.00	N/A
11	16.00	N/A
12	15.50	N/A
13	13.00	N/A
14	10.00	N/A
15	9.00	N/A
16	8.00	N/A
17	7.00	N/A
18	2.00	N/A

[18] Note that another equilibrium exists in which the person with $V_T = 29$ travels the bridge instead of the person with $V_T = 30$.

3.4. Efficiency with Heterogeneous Values of Time

In this section, the values of time are again allowed to differ across subjects. In a discrete case, there is no simple way to generalize the minimum total time cost concept without knowing the distribution of values of time.[19] For example, assume that 18 people are traveling on the same route grid and the parameters for travel time are the same as in the discrete example from Section 3.3. If the person with the highest value of time has V_1 = $1,000 per hour, while everyone else has a value of time of $10 per hour, the efficient outcome involves only the high-value person on the bridge. This is because the MEC from the second person traveling the bridge is higher than the MPB.[20]

Formally, an efficient outcome occurs when the following problem is solved:

$$(10)\quad \text{Min} \sum_{i=1}^{18} V_i t_j$$

subject to $t_B = \alpha + \beta \mathrm{T}$,

with $j \in (B,H)$, and t_H, α and β are given. To help solve this problem, note that for a given number of subjects traveling the bridge (as long as the travel time on the bridge is shorter), the lowest total time cost occurs when the subjects with the highest values of time travel the bridge.

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[19] Remember that tolls are transfers here.

[20] Specifically, if only the person with the highest value of time travels the bridge, her travel time is 28.6 minutes. With the addition of another driver, the high value-of-time driver's time increases by 0.2 minutes, leading to a MEC of $3.33 at a rate of $1,000 per hour. The MPB of the second driver on the bridge is 1.2 minutes, or $0.20 at a rate of $10 per hour.

Since the people with high values of time will travel the bridge, the shapes of curves in Figure 3 can be determined, which generalizes from Figure 2 to a heterogeneous case. From Equation (6), the MEC increases by a smaller amount for each additional driver, while from Equation (2), the MPB also flattens as the number of drivers increases. Based on these conclusions, both the MEC and MPB curves become flatter as *T* increases. As in the homogeneous value of time case, Equation (10) is minimized when the MEC and MPB curves intersect.

A Discrete Example

To get an efficient outcome, we now know that the people with the highest values of time will travel on the bridge. Using the same example as in Section 3.3 (with values of time seen in Table 1), Table 2 shows the total travel time cost of all drivers as a function of the number of people traveling the bridge, assuming that the people with the highest values of time travel the bridge. The table shows that total travel time cost is the same if either nobody or eight people travel the bridge, since everybody's travel time is 30 minutes. This cost is lower when any number of travelers from one to seven travels on the bridge, because the people that travel the bridge require less than 30 minutes to travel the bridge.

In order to see that three people traveling the bridge is optimal, compare the marginal private benefit against the marginal social cost for the third and fourth drivers on the bridge. In the case of the third driver, the time for this driver is reduced by 1.0 minute, from 30 to 29 minutes. The person with the fourth highest

Figure 3: Marginal external cost (MEC) and marginal private benefit (MPB) in a heterogeneous value of time case

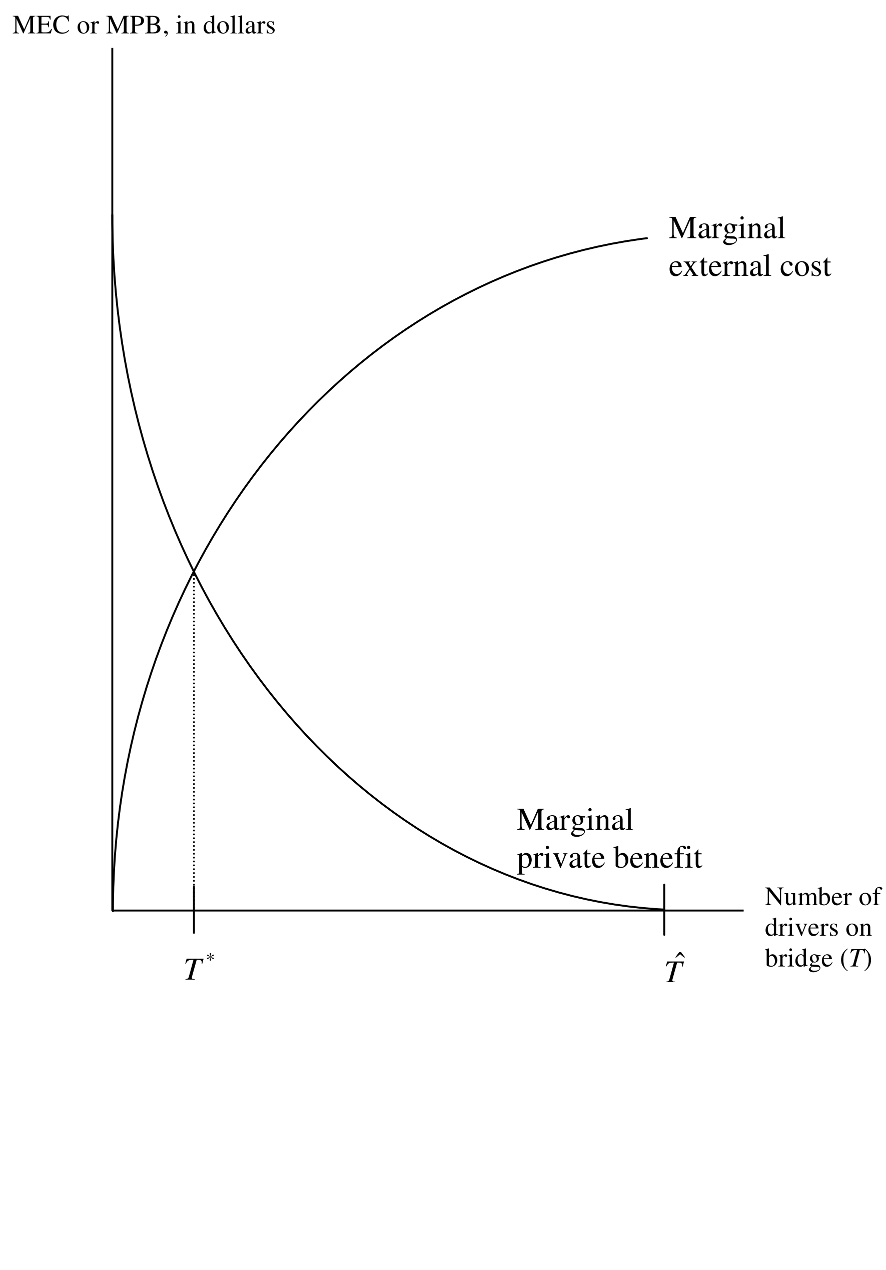

Table 2: Total travel cost as a function of number of travelers on the bridge, using the same distribution of drivers as in Table 1

Number of drivers on bridge	Travel time on bridge, in minutes (t_B)	Total travel time cost of all drivers, in dollars
0	28.4	261.25
1	28.6	258.92
2	28.8	257.65
3	29.0	257.08
4	29.2	257.25
5	29.4	257.95
6	29.6	258.86
7	29.8	259.98
8	30.0	261.25
9	30.2	262.66
10	30.4	264.20
11	30.6	265.83
12	30.8	267.56
13	31.0	269.36
14	31.2	271.18
15	31.4	273.05
16	31.6	274.94
17	31.8	276.87
18	32.0	278.67

value of time is at $70 per hour, which translates to a MPB of $1.17 if this driver moves from the highway to the bridge. However, external costs are imposed on the two drivers already on the bridge, with an increase in travel time of 0.2 minutes on those already driving on the bridge when the third driver is added. Assuming that the drivers with the highest values of time are already on the bridge, the MEC is 33 cents for the driver with the highest value of time and 27 cents for the second highest. Thus, adding the third driver to the bridge is more efficient, since the MPB of the fourth driver exceeds the MEC of the three drivers already on the bridge. A similar analysis of adding the fourth driver on the bridge shows that the total MPB is less than the MEC.

3.5. Travel Time Uncertainty and Mixed-Strategy Nash Equilibria

There is one important item to note regarding equilibria. In previous subsections, each person knows the number of other travelers who take the bridge with certainty. In reality, decisions of others are not known until after each repetition is over. This may lead to subjects favoring mixed strategies over pure ones. Mixed strategies, including those that are Nash equilibria, are analyzed more in Section 5.3.1.

4. Experimental Design

The two-route travel grid from the Pigou-Knight-Downs paradox is used in the two experiments described here. In the first experiment, two parts (“segments”) incorporate tolls, each of which predict lower travel time costs for the population. In the last segment of the first experiment, subjects must evaluate a situation in which subjects have the opportunity to give up money to reduce actual waiting time. During this waiting time,subjects sit in a computer lab and do nothing. In this case, theory predicts subjects with a high enough value of time will pay the toll to reduce waiting time.

Similar to the last segment of the first experiment, the second also incorporates subject heterogeneity. However, in contrast to Experiment 1, in Experiment 2 the time costs are monetized, and subjects are given one of five profiles, where each profile has a different monetary deduction per minute of travel time. Although monetized time costs are less realistic than actual time costs, they allow for the prediction of all equilibria.

In both experiments, each subject may stay on the same route or change routes from round to round, but no one is permitted to change their choice within a round once their decision is made. At the end of each round, subjects receive information as to how many people travel on the bridge in that round, along with the total point deduction. Each experiment lasts about one hour, and subjects earn an average of about $15 for participation in one of the experiments.

4.1. Experiment 1

In each experimental session, 18 subjects[21] travel from point A to point B using either a congested bridge or an uncongested highway in each round (see Figure 1).[22] The highway guarantees a travel time of $t_H = 20$ minutes. In contrast, while the bridge is uncongested for the first traveler, and hence has a travel time of 10 minutes, each additional driver on the bridge adds one minute to every bridge user's travel time.[23] In other words, if there are T subjects traveling the bridge in any round, the travel time for each person is $t_B = (9 + T)$ minutes, or $\alpha = 9$ and $\beta = 1$ using the notation from the previous section.

Each of the 10 experimental sessions consists of three segments with 20 rounds (or repetitions) each, and each subject begins with 8500 points and a $5 show-up fee.[24] Points are deducted for travel time in the first two segments, but not for the third segment. Instead of paying a monetary equivalent for time in Segment 3, subjects are told that more travel time within this segment results in an increased physical waiting time before receiving their payment at the end of the experiment. Finally, tolls are charged (in the form of point deductions) on the bridge in the final

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[21] Due to non-participation one group had only 17 subjects, but for simplicity all discussion will assume 18 subjects in all sessions.

[22] The experiment was programmed and conducted with the software z-Tree (Fischbacher forthcoming).

[23] Note that if all subjects travel the bridge, each subject requires 27 minutes in travel time.

[24] Subjects receive the show-up fee even if all points are used up. In no case does any subject lose all points in this experiment.

two segments of the experiment. After the experiment is finished, the remaining points are converted at a rate of 50 points per $1.[25]

In Segment 1, subjects are told that each minute of travel time leads to a 10-point deduction, but no tolls are charged.[26] If subjects are profit maximizers, they attempt to choose the route that minimizes their point deduction in every round. This, of course, means that their route choice depends on their expectations about what the other 17 subjects will do in any particular round. Within this framework, theory predicts an equilibrium with 20 minutes of travel time on both routes in Segment 1. This occurs when $T = 11$.

In Segment 2, subjects continue to pay a 10-point deduction per minute of travel time, but now there is a 60-point per round toll charge.[27] At a cost of 10 points per minute, a 60-point toll translates to the equivalent of six minutes of travel time cost. This means that a 14-minute commute on the bridge is now equivalent (in total point deductions per round) to a 20-minute commute on the highway. So the new toll equilibrium results in a drastic decline in the number of subjects on the bridge, with five people using the bridge, compared to 11 in Segment 1.[28] This

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[25] All subjects thus start with a $170 endowment.

[26] Any subject that uses the highway in every round of Segment 1 results in a total of 4000 points deducted, 20 rounds at 20 minutes each, times 10 points per minute of travel time. The same total point deduction also occurs for any subject whose average travel time in Segment 1 is 20 minutes.

[27] To prevent unneeded complication in the experiments described in this paper, tolls are not refunded to subjects.

[28] Based on the theory section, the optimal toll is based on the equivalent of 5.5 minutes, or 55 points. This would result in a prediction of 5.5 travelers on the bridge. Since fractional numbers of

equilibrium also minimizes the total travel time of all subjects. Since travel costs are homogeneous the Segment 2 equilibrium is also efficient.

In the final part of this experiment, subjects can trade-off money for waiting time. A subject only pays a 6-point toll charge to use the bridge in Segment 3, but no longer faces a point deduction for travel minutes in the experiment. Instead of a point deduction for travel time, a subject's sum of travel minutes for the 20 rounds in this segment (denoted as *N*) is converted into waiting time at the end of the session. The amount of time that subjects must wait before being paid is $\frac{N-200}{10}$ minutes. Within this set-up, if there are fewer than 11 bridge users in any round, then subjects can literally give up money to gain additional time.

Based on the above conversion factor, each minute of reduced travel time in Segment 3 results in a 0.1-minute reduction of actual waiting time. This means that the actual waiting time saved each time the bridge is traveled (versus traveling the highway) is not $[t_H - (\alpha + \beta T)]$, but $\frac{[t_H - (\alpha + \beta T)]}{10}$. Thus, similar to the theory presented in Section 3,

$$(11)\quad C_T = 60C \Big/ \left[\frac{t_H - (\alpha + \beta T)}{10} \right].$$

travelers are not allowed, two optimum results can occur, with either five or six travelers on the bridge.

Since C =\$0.12,[29] $t_H = 20$, $\alpha = 9$, and $\beta = 1$, then

(12) $C_T = \$7.20 \Big/ \left[\frac{11-T}{10}\right]$,

where $\frac{11-T}{10}$ is the number of minutes saved on the bridge versus traveling the highway in a particular round. Table 3 shows the relationship between T, the number of minutes reduced for each round the bridge is traveled in Segment 3 (as a function of T), and C_T.

Table 3: Cost savings per hour based on the number of bridge travelers, Experiment 1

Number of travelers on the bridge (T)	Waiting time reduction each round the bridge is traveled	Cost to save one hour of waiting time at this rate (C_T)
1	1.0	\$7.20
2	0.9	\$8.00
3	0.8	\$9.00
4	0.7	\$10.29
5	0.6	\$12.00
6	0.5	\$14.40
7	0.4	\$18.00
8	0.3	\$24.00
9	0.2	\$36.00
10	0.1	\$72.00
11	0.0	Undefined[30]

[29] Six points at a conversion rate of 50 points per dollar is equivalent to \$0.12.

4.2. Experiment 2

In each experimental session, 15 subjects travel the same route network from point A to point B. Subjects are told that the highway has a guaranteed t_H = 25-minute travel time, while the bridge's travel time is $t_B = (11 + T)$ minutes. A toll of 70 points is charged in each round a subject travels the bridge in this experiment. There are two segments in this experiment of 50 rounds each. Subjects are each given their specific point deductions on their computer screen once the verbal instructions are given. In each segment of each session, there are five sub-groups of three people each. There are three possible formats of the sub-groups:

- (I) One sub-group each with a point deduction of 4, 7, 10, 13, and 16 points per minute (high heterogeneity)
- (II) One sub-group each with a point deduction of 8, 9, 10, 11, and 12 points per minute (low heterogeneity)
- (III) A control group with all sub-groups having point deductions of 10 points per minute (no heterogeneity)

There are three experimental designs from the three combinations of formats:

- (A) Segment 1 with Format I and Segment 2 with Format II
- (B) Segment 1 with Format II and Segment 2 with Format I
- (C) Both segments with Format III

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[30] Cost of time is undefined here because traveling the bridge does not reduce waiting time any.

In this experiment, the number of points that each person starts with is dependent on the point deductions per minute for each of the two segments, but each subject receives a $5 show-up fee. The point endowment information is displayed in Table 4.[31] Since all costs are monetized in this experiment, subjects do not incur a waiting time at the end of the experiment.

Table 4: Point endowments for subjects in Experiment 2

Point deductions per minute, Segment 1	Point deductions per minute, Segment 2	Initial point endowment
4	12	20000
7	11	22850
8	16	28600
9	13	27150
10	10	25700
11	7	22850
12	4	20000
13	9	27150
16	8	28600

To reach an equilibrium similar to what the Pigou-Knight-Downs paradox predicts, I look most closely to the sub-group with a 10 point per minute deduction.

[31] Six subjects lost all of their points by the end of the experiment. However, I do not suspect that the results are substantially affected, because in all but one case, all subjects had a positive number of points after Round 97 of 100. One subject lost all remaining points in the 91st round. This person has a 4 point per minute deduction in Segment 2, and travels on the highway in each of the final 12 rounds of the experiment.

Recall from the theory in Section 3.3, that when $C_T = V_T$, the T^{th} person travels on the bridge. With this assumption, all three formats have equilibrium of seven subjects on the bridge. In Formats I and II, the two sub-groups with highest point deductions all travel on the bridge, along with one person with a 10 point deduction per minute. There are other Nash equilibria that are possible, due to the discrete nature of the experiment, but they are examined minimally in this paper. These equilibria are derived in Appendix B.

5. Experimental Results and Analysis, Experiment 1

5.1. Data Summary and Comparison to Pure-Strategy Equilibria

Table 5 reports the average number of bridge travelers per round for each of the experimental groups, with column 2 reporting the average number of bridge travelers in the no-toll case (Segment 1), column 3 reporting the results for the toll case (Segment 2), and column 4 reporting the results for the heterogeneous time cost case with tolls (Segment 3). Consistent with the theory presented in Section 3, tolls persuade some subjects to change their route choice from the congested bridge to the uncongested highway. Specifically, about five fewer subjects travel the bridge on average in Segments 2 and 3 than in Segment 1. This means that an optimal toll is a successful tool to re-route traffic into a more efficient equilibrium.

Subjects' waiting time is based in part on their route choices in the 20 rounds of Segment 3. On average, each subject travels the bridge 6.1 times, with an average

Table 5: Average number of bridge travelers per round in each segment, by session, Experiment 1

	Segment 1	Segment 2	Segment 3
Session 1	10.70	5.85*	6.35
	(2.15)	(1.79)	(1.69)
Session 2	11.10	5.60	5.00
	(2.05)	(2.09)	(1.56)
Session 3	9.90	5.55	6.50
	(2.45)	(2.28)	(1.36)
Session 4	10.90	5.15	6.95
	(2.83)	(1.31)	(1.90)
Session 5	11.05	5.70	5.00
	(1.93)	(2.43)	(1.59)
Session 6	11.30	5.80*	5.30
	(2.77)	(1.51)	(1.49)
Session 7	10.85	5.70	5.90
	(1.98)	(1.66)	(1.71)
Session 8	10.85	6.00*	5.00
	(1.69)	(2.03)	(1.30)
Session 9**	10.80	5.60	4.20
	(1.51)	(1.96)	(1.79)
Session 10	11.35	5.50	4.70
	(2.50)	(1.93)	(1.63)
All sessions	10.88	5.65*	5.49
	(2.21)	(1.89)	(1.79)

Standard deviations are in parentheses.

* Denotes significantly different from 5 at the 5% level in Segment 2. However, the individual sessions are not significantly different from 5 if only the last 10 or 15 rounds are examined. Also, the all sessions average is not significantly different from 5 if the last 10 rounds are examined.

** Only 17 subjects participated in this session.

Table 6: Distribution of number of times the bridge is traveled by each subject in Segment 3, Experiment 1

Number of bridge trips	Fraction of subject pool (out of 179 subjects)
0	19.6%
1-4	27.9%
5-9	27.9%
10-14	11.7%
15-19	8.9%
20	3.9%

Table 7: Distribution of waiting times, Experiment 1

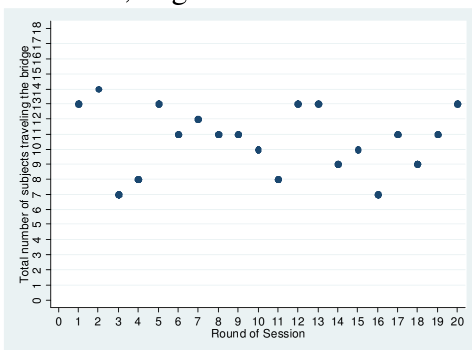
Waiting time, in minutes	Fraction of subject pool (out of 179 subjects)
6-7.9	0.6%
8-9.9	2.2%
10-11.9	6.1%
12-13.9	7.8%
14-15.9	13.4%
16-17.9	21.2%
18-19.9	29.1%
20	19.6%

waiting time of 17.0 minutes. Tables 6 and 7 report the distributions of the number of times that each subject travels the bridge and the subjects' waiting times, respectively. Table 6 shows that only about a quarter of the subjects travel on the same route in each round of Segment 3, while the rest travel both routes at least one time each. Table 7 shows that the waiting times range from 7.4 to 20 minutes. The median waiting time of all subjects is 17.8 minutes. Note that 60 percent of subjects wait 16 minutes or more, implying that they are not willing to give up much money to reduce their waiting time after the end of the session. However, about 9 percent reduce their waiting time to below 12 minutes. These subjects show behavior of being willing to pay the toll and travel the bridge most of the rounds in order to reduce their waiting times substantially.

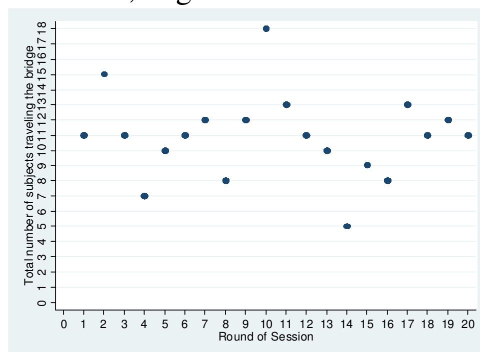
For Segment 1, recall that the theory in Section 3 predicts 11 subjects on the bridge and 7 on the highway in pure-strategy equilibrium. All of the session averages are within 1.1 of this prediction, with none of these averages statistically differing from 11. Round-by-round results can be seen Figure 4, while Figure 5 shows the nearly normal distribution of the number of travelers on the bridge. As seen in Figure 4, the number of people traveling the bridge often changes after a round is in equilibrium. Although no single subject can be made better off by being the only person to switch routes after equilibrium is reached, some people tend to switch after a round in equilibrium. Session 3 is a good example. Despite the fact that this group has reached equilibrium in Round 3, in the fourth round, two subjects switch from the bridge to the highway, while six switch from the highway to the bridge, resulting in 15 subjects on the bridge. With nearly half of the subjects switching routes after equilibrium is reached, predictions made by a pure strategy

Figure 4: Round-by-round results of number of travelers on the bridge for each session and segment, Experiment 1

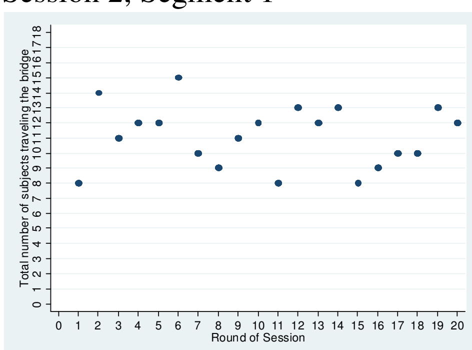
Session 1, Segment 1



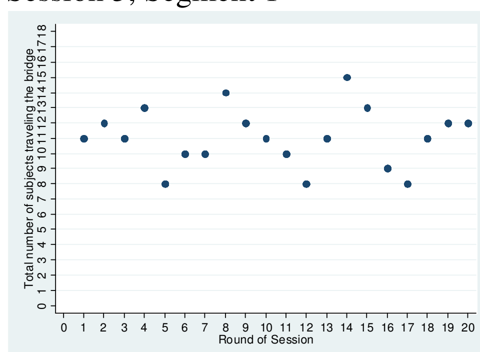
Session 4, Segment 1



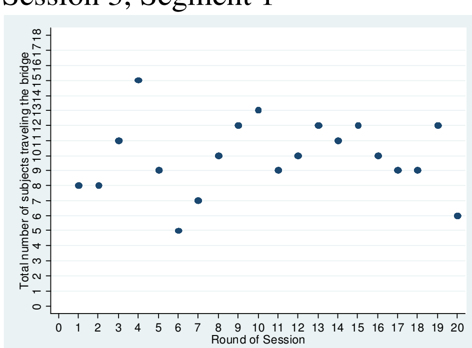
Session 2, Segment 1



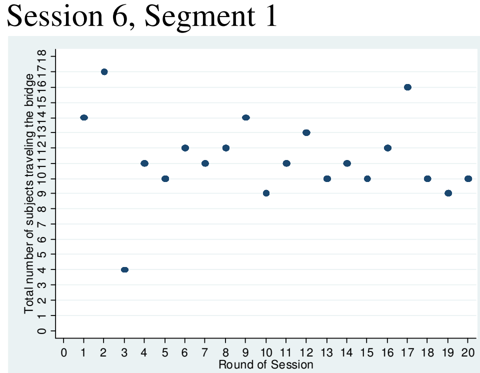
Session 5, Segment 1



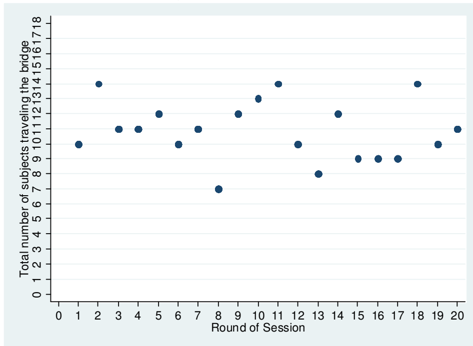
Session 3, Segment 1



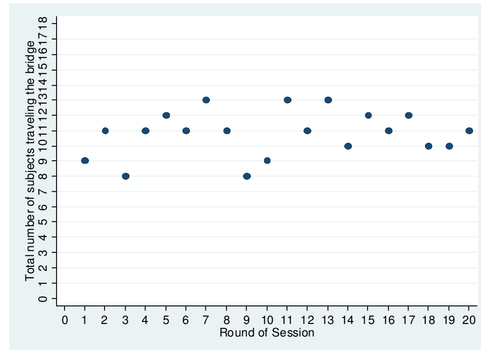
Session 6, Segment 1



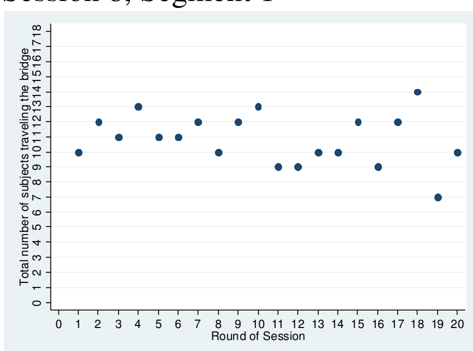
Session 7, Segment 1



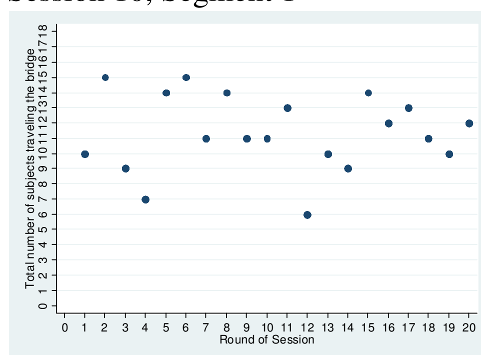
Session 9, Segment 1



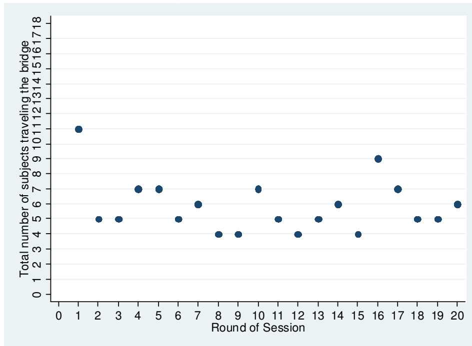
Session 8, Segment 1



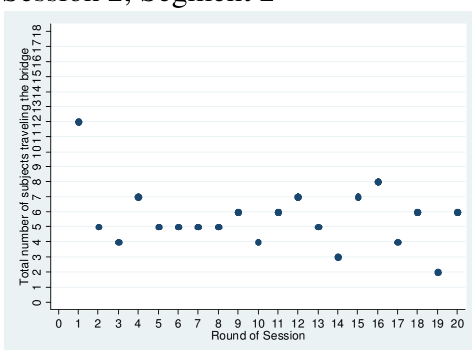
Session 10, Segment 1



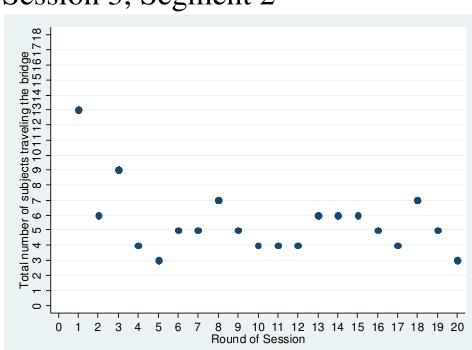
Session 1, Segment 2



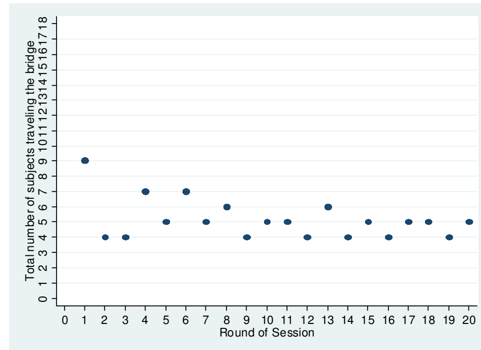
Session 2, Segment 2



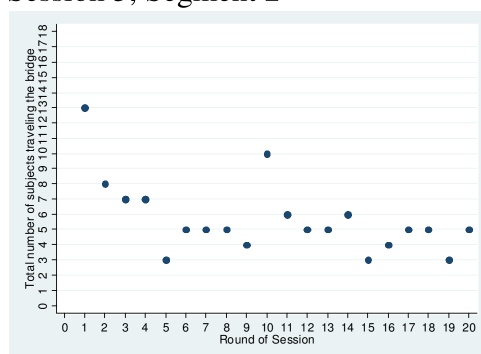
Session 3, Segment 2



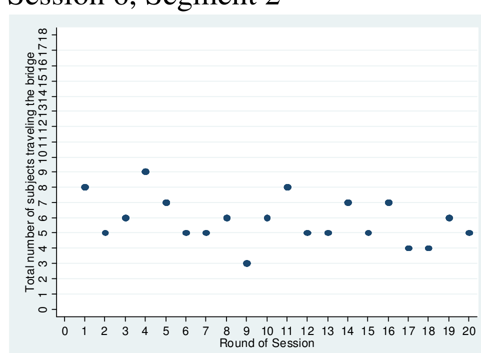
Session 4, Segment 2



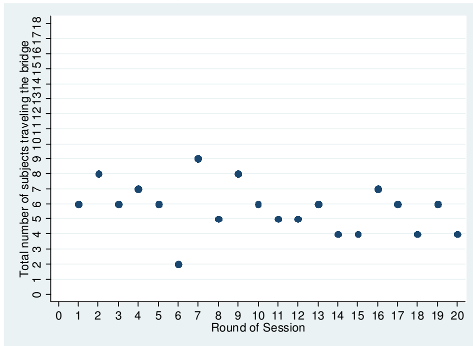
Session 5, Segment 2



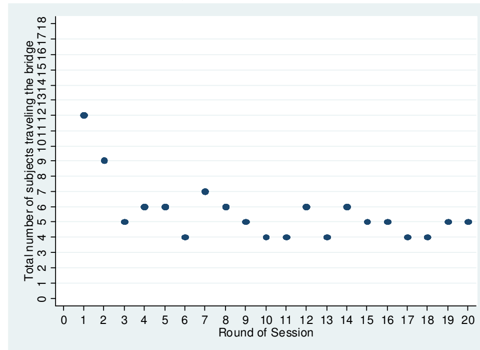
Session 6, Segment 2



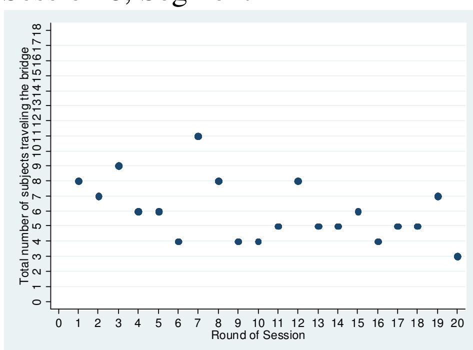
Session 7, Segment 2



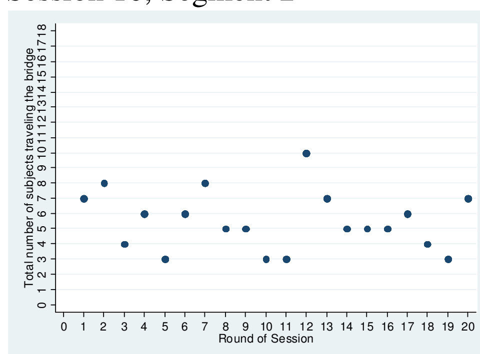
Session 9, Segment 2



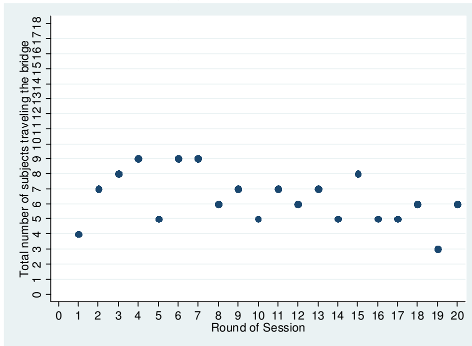
Session 8, Segment 2



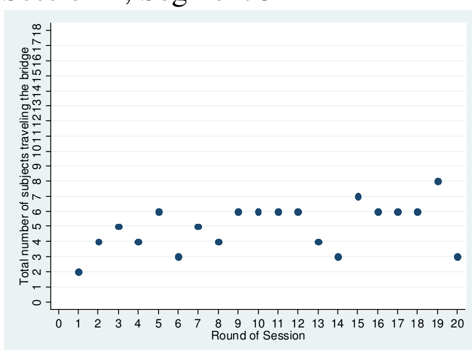
Session 10, Segment 2



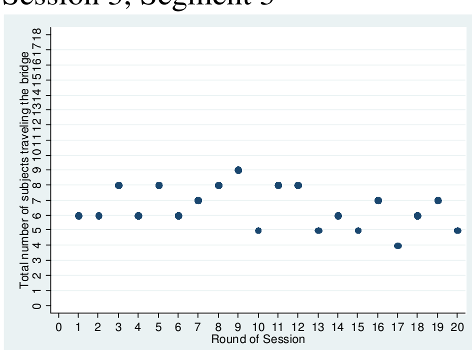
Session 1, Segment 3



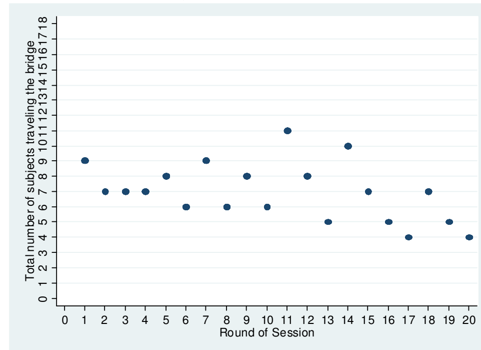
Session 2, Segment 3



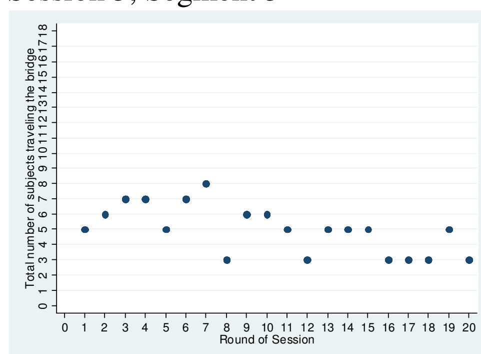
Session 3, Segment 3



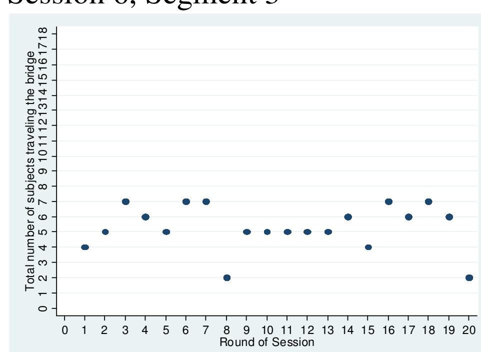
Session 4, Segment 3



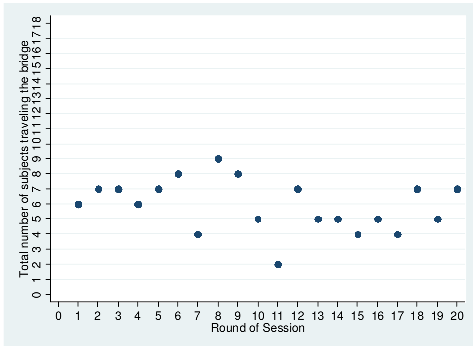
Session 5, Segment 3



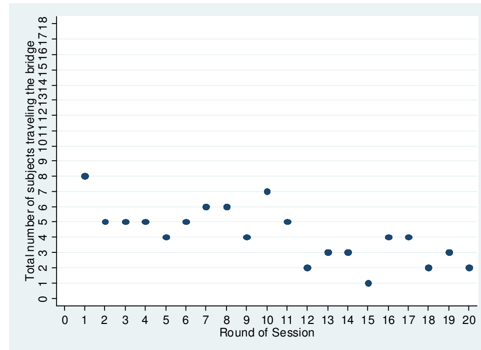
Session 6, Segment 3



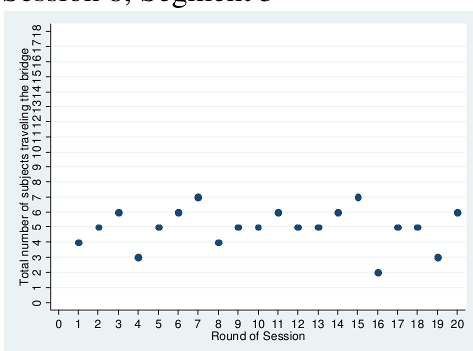
Session 7, Segment 3



Session 9, Segment 3



Session 8, Segment 3



Session 10, Segment 3

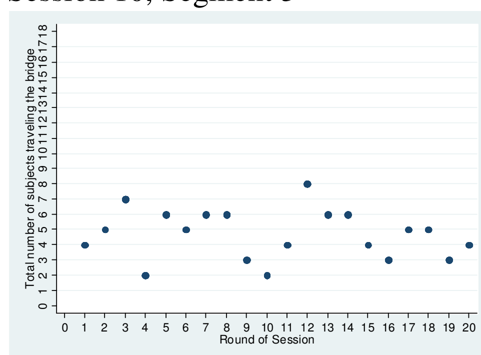
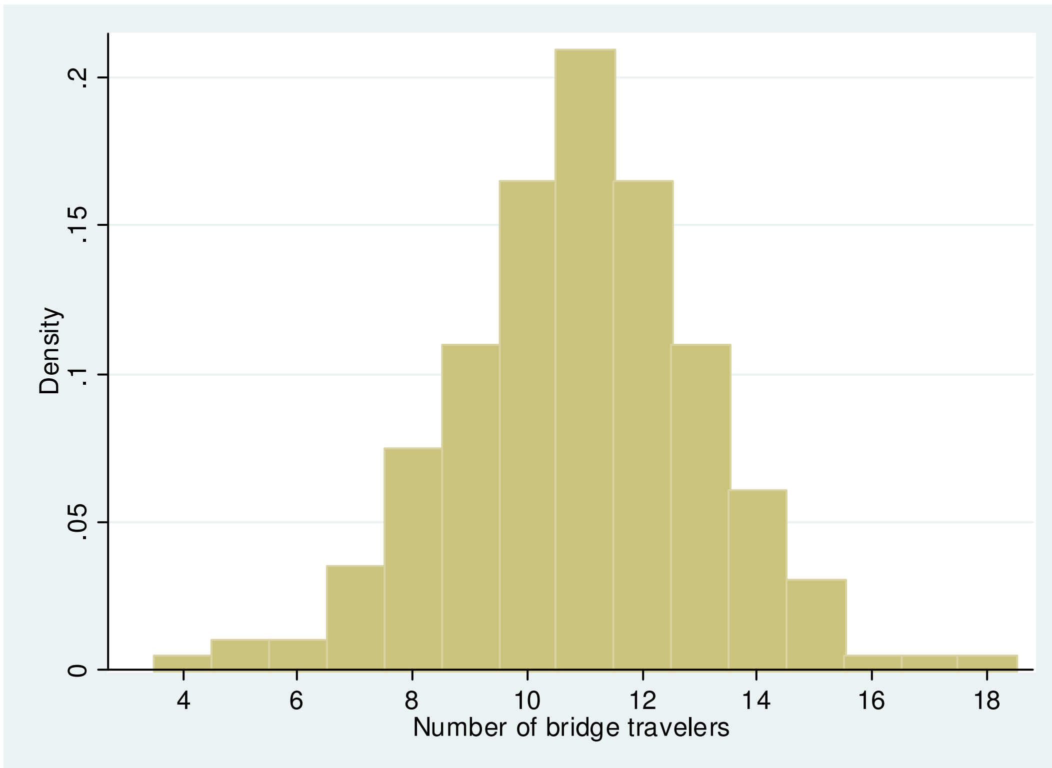


Figure 5: Distribution of number of bridge travelers in Segment 1 of Experiment 1, by round



Nash equilibrium typically do not apply in such a situation. Subjects may also lack full rationality, although testing this is difficult since subjects may be doing what they think is optimal given the actions of others.[32]

In Segment 2, theory predicts 5 subjects on the bridge and 13 on the highway in equilibrium.[33] Unlike Segment 1, some of the group averages significantly differ

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[32] For more on limited rationality, see Arthur (1994).

[33] Note that 4 subjects on the bridge is also an equilibrium, but the results show that this equilibrium has little affect on actual subject behavior.

from this equilibrium. Specifically, Sessions 1, 6, and 8, along with the collective average of all of the groups, significantly average more than 5 subjects on the bridge per round. This is likely due to a transitioning effect going on from the end of the first segment to the beginning of the second, where subjects may not initially understand the new environment. For example, the first four rounds of Segment 2 for Session 5 result in 13, 8, 7, and 7 bridge travelers, respectively. In the fifth round, the number of bridge travelers is finally below equilibrium for the first time, and equilibrium is finally reached for the first time in the sixth round. If the first 10 rounds are removed from each session of Segment 2, none of the means for this part of the experiment is significantly different from 5.[34] As in Segment 1, many of the rounds are out of equilibrium after equilibrium is reached for the first time.

Finally, equilibrium in Segment 3 depends on the distribution of the values of time within each group of subjects. However, the results of the experiment suggest some general statements about subjects' values of time. In each of the groups, most of the later rounds in the segment have two to seven bridge travelers. From Table 3, the per-hour cost of traveling the bridge can be examined based on the number of bridge travelers. In the case of a round of two bridge travelers, the per-hour cost to use the bridge is $8, while if seven subjects travel the bridge, the per-hour cost is $18. Thus, from Table 3, any round in which the number of subjects ranges from two to seven on the bridge, a subject wants to travel the bridge if their value of time

[34] If only the first five rounds are removed, the same result holds, except that the average for all sessions is significantly above 5.

is more than $18 per hour. A subject in this situation never wants to travel the bridge if their value of time is less than $8 per hour. Subjects with values of time between $8 and $18 per hour may or may not want to travel the bridge, depending on what the outcome is. Since I would suspect that most of the subjects in this experiment have values of time in the $8-$18 range, 20 rounds may not be enough for each subject to determine which route to travel on in equilibrium. This topic is looked at more formally in Section 5.4, which further examines the issue of value of time.

5.2. Efficiency

Given the imposed constant value of time in Segments 1 and 2, minimizing total travel time is equivalent to an efficient outcome. As such, social welfare on the route grid used in this experiment can be measured by comparing the average travel times between the no-toll and toll schemes. Although drivers perceive tolls as costs, they are simply transfers if they go to government. So the lower the average travel time is, the higher the social welfare is for a group with homogeneous travel costs. Thus, the lower the overall travel time, the more efficient the outcome. Given the imposed homogeneity in Segments 1 and 2, comparing average travel times tells something about relative efficiency. Based on the model described in Section 3.2, for a given group of drivers, five or six drivers on the bridge will yield the fewest total number of minutes traveled.[35] This results in the minimum possible total travel time of 330 minutes.[36] With the 10 sessions of the experiment, the best attainable average time per round is 18.32 minutes.[37] In Segment 1, the average is 20.20 minutes, or 10.2% higher than the efficient outcome. In fact, only 2% of the rounds achieved the efficient outcome.[38] In Segment 2, the average travel time is 18.51 minutes, or 1.0% more than the efficient outcome. Here, 48.5% of the rounds resulted in efficient outcomes.

Although values of time for individuals are not known in Segment 3, comparing the results to Segment 2 gives us insight about how travel times compare between a tolling scheme with homogeneous profiles and a tolling scheme with people having different values of time. In Segment 3, the average travel time is 18.49 minutes, or 0.9% higher than the minimum possible total travel time. Further, the total travel time is minimized in 46% of the rounds. Note that although theory predicts an efficient outcome in equilibrium in Segment 2 but is unclear as to what equilibrium is in Segment 3, Segment 3 averages the lowest travel time. However, Segment 2 has more rounds with total travel time minimized.

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[35] Recall that the actual calculation results in 5.5 subjects on the bridge, but fractional travelers are not possible.

[36] This assumes 18 subjects. In the one group with 17 subjects the minimum total travel time is 310 minutes.

[37] This average factors in that one group has 17 subjects.

[38] An efficient outcome is defined as when five or six subjects travel the bridge, since total travel time is minimized.

5.3. Analysis

5.3.1. Symmetric Mixed-strategy Equilibria

In most of Section 3, the discussion focuses on situations in which subjects know with certainty the actions of the other players before making her or his decision. In reality, this does not occur, implying that some or all subjects may decide route choice based on mixed strategies. More evidence supporting the possibility of subjects playing mixed strategies comes from Figure 4. In this figure, once a pure-strategy Nash equilibrium is reached at any point in the first two segments, one or more subsequent rounds in the same segment are typically not in equilibrium.

Appendix A derives a symmetric mixed-strategy Nash equilibrium in Segment 1, with each person playing bridge with probability 10/17. Also from Appendix A, the probability of playing bridge in mixed-strategy equilibrium is 4/17 for Segment 2. Note that in mixed-strategy equilibrium for Segment 1, the expected number of bridge travelers is $18 \times (10/17)$, or 10.59, which is less than in the pure strategy Nash equilibrium prediction. In Segment 2, the expectation is $18 \times (4/17)$, or 4.24 travelers on the bridge, also less than the pure strategy Nash equilibrium. In Appendix A, I show that the total number of travelers on the bridge in Segment 1 is not significantly different from the mixed-strategy prediction at the 5% level.[39] However, in Segment 2 of Experiment 1, the total number of travelers on the bridge

[39] In Selten *et al* (2007), a similar calculation in the experiment rejects the null hypothesis that the number of travelers on the bridge each round is consistent with the mixed-strategy Nash equilibrium.

is significantly different from the mixed-strategy prediction. Another measure that can be compared between experimental results and the mixed-strategy prediction is the number of road changes within a segment. As Appendix A shows, there are far fewer route changes in Segments 1 and 2 than the mixed strategy predicts.[40] So although there may be some players acting close to the mixed-strategy equilibrium, there appears to be some route "stickiness," in which once a player is on a particular route, there is an increased tendency to stay on the route in the next round.[41]

5.3.2. The Impact of Subject Characteristics on Decision-Making

In Segment 3, route choice is determined by the trade-off between time and money. As such, it is possible that personal characteristics, such as those listed in Table 8, may play an important role in determining the route chosen by each subject. However, the data do not generally support this conjecture. The average number of bridge trips does not differ across subject characteristics self-reported by subjects before the experiment. In particular, t-tests for each of the following characteristics fail to reject the null hypothesis of no difference in the average number of times the bridge is used, by subject: whether the subject is a male, a graduate student, a

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[40] The similar calculation for the Selten *et al* (2007) experiment is also rejected in their experiment.

[41] Walker and Wooders (2001) show that top players of a game may behave more consistently to mixed-strategy equilibria than inexperienced players. Despite this observation, they also conclude that top tennis professionals switch their serve direction more often than random play would predict. Other papers show that mixed strategies may not be able to explain individual behavior well, but the same mixed strategies can be consistent with aggregate behavior. Some examples are by Rapoport *et al* (2004), Levine and Palfrey (2007), and Müller and Schotter (2007).

Table 8: Summary statistics of subjects' personal characteristics, Experiment 1

Characteristic	Mean	Standard deviation
Male	0.44	0.49
Age	20.68	3.03
Subject is a student at UCSB	0.98	0.13
Number of units enrolled in (conditional on being a student)	14.86	2.79
Freshman	0.27	0.45
Sophomore	0.19	0.39
Junior	0.27	0.45
Senior	0.18	0.39
Graduate student	0.09	0.29
Grade point average 3.5 or higher[42]	0.33	0.47
Grade point average below 2.5	0.04	0.20
Works at least five hours per week	0.36	0.48
Earns $10 per hour or more	0.17	0.37
Earns $12 per hour or more	0.11	0.32
Earns $15 per hour or more	0.08	0.28
At least one parent has a bachelor's degree	0.71	0.46

[42] Grade point average information is missing from two students.

freshman, a sophomore, a junior, a senior, has a grade point average (GPA) of 3.5 or higher, has a GPA below 2.5, has a job of at least five hours per week, has at least one parent with a bachelor's degree, earns $10 per hour or more, earns $12 per hour or more, earns $15 per hour or more, is age 21 or older, is enrolled in 15 units or more at the time. Thus, no one characteristic by itself seems to have an effect on the frequency of bridge travel in Segment 3.[43]

As seen above, one of the most surprising findings is that route selection appears to be independent of subjects' wages. There does not appear to be muchdifference between populations, whether the characteristic is earning $10, $12, or $15 or more per hour,[44] or whether or not employed. This result may seem counter-intuitive, because a person's wage is often used as an approximation of one's value of time (see Deacon and Sonstelie (1985), for example).

Among the subject pool, the students with less educated parents are most likely to face time constraints, since they are more likely to be working in order to afford attending college. This time working limits leisure time.[45] It is interesting, then, to compare the Segment 3 route choices of working students with neither parent having an undergraduate degree with those of the other subjects. Table 9

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[43] Surprisingly, there are two significant differences of number of bridge trips in Segment 1. Graduate students average 7.88 trips per person in Segment 1, while others average 12.56 trips per person. Sophomores average 14.12 trips, while the rest averaged 11.68. No significant differences are seen in Segment 2.

[44] The same holds true whether people without a job are included or excluded in the analysis.

[45] Many studies have been done looking at educational outcomes of low-income college students. For example, see Kiker and Condon (1981), Ehrenberg and Sherman (1987), Taubman (1989), and Dynarski (2004).

shows results in Segment 3 controlling for the possible combinations of whether or not someone has a job of five hours or more per week and at least one parent with a bachelor's degree. Those working and having no parent with a bachelor's degree are significantly different than the other subjects in average number of bridge trips, with clustered[46] probit regressions leading to p-values in the 0.030 to 0.054 range.[47] With similar ordinary least square regressions, subjects working and having no parent with a bachelor's degree are about 18 to 20 percentage points more likely to travel the bridge in Segment 3, and regression p-values ranging from 0.007 to 0.025.[48] These 17 subjects averages 9.71 bridge trips, while the other population subsets average 5.65 to 5.86 trips. Although the group traveling the bridge most often consists of 9.5% of the subject population, 41% of the subjects that travel the bridge at least 15 of the 20 rounds are in this group.

5.3.3. Variation in Number of Bridge Travelers by Group

Theory predicts the same equilibrium for each group in Segments 1 and 2, specifically 11 and 5 subjects on the bridge, respectively. However, due to potential

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[46] Regression results are clustered by individual.

[47] When the interaction variable of those working and a parent without a bachelor's degree is the only variable in the regression, the p-value is 0.030. When all possible combinations except for the control of having a job and a parent with a bachelor's degree are included, the p-value is 0.037. When controls for gender, age, number of units currently taken, graduate student, GPA above 3.5, and GPA below 2.5 are included, the p-values increase slightly, to 0.054 and 0.051, respectively.

[48] Using the same sets of controls as in the clustered probit regressions, the respective coefficients and p-values are 0.197 and 0.007, 0.203 and 0.013, 0.184 and 0.020, and 0.194 and 0.025.

Table 9: Average number of trips on the bridge in Segment 3 of Experiment 1, based on whether the subject is working, and whether either of the subject's parents has at least a bachelor's degree

Characteristics of subject	Works at least five hours per week	Does not work at least five hours per week
Has at least one parent with a bachelor's degree	5.65 (5.24) [48]	5.78 (5.41) [79]
Has neither parent with at least a bachelor's degree	9.71 (7.87) [17]	5.86 (5.82) [35]

Notes: Standard errors are in parentheses.

Number of subjects in the category is in brackets.

heterogeneity in value of time, equilibrium can vary from group to group in Segment 3. Group-to-group heterogeneity can be tested for all three segments using an F-test. More specifically, the null hypothesis is that the mean number of bridge travelers is the same across sessions. For comparison, F-tests are also provided for Segments 1 and 2, segments that should provide no significant differences in means.

In Segment 1, the average number of bridge travelers per round by group ranges from 9.90-11.35. (See data summary in Table 5 for a more thorough summary.) Using an F-test for the difference in means, these averages are not statistically different from each other, with a p-value of 0.743. This is consistent with the theoretical prediction of the same equilibrium in each group. Segment 2's averages by group range from 5.15-6.00. Again, the averages are not significantly different from each other, with a p-value of 0.978. This is also consistent with the theoretical prediction of the same equilibrium in each group. Segment 3's averages by group range from 4.20-6.95. These averages are significantly different, with a p-value of less than 0.001. This suggests that heterogeneity in the value of time between groups contributes to differential outcomes by group in Segment 3, while experiments with monetized values of time do not necessarily exhibit this characteristic.

5.4. The Value of Time

Segments 1 and 2 deal with decisions made for monetary purposes only whereas real commuting decisions typically involve some combination of monetary and time costs. Monetary costs may include fuel, tolls, train or bus fare, and a fraction of long-term maintenance of a vehicle. This section deals with the time costs of commuting behavior, and also more closely looks at the values of time of the subjects in Experiment 1.

There is substantial existing work regarding value of time decisions. Becker (1965) assumes constant marginal costs for commuting and makes other assumptions to conclude that commuting time increases when income goes up only when income elasticity of demand for space exceeds one. Leclerc, Schmitt, and Dube (1995) examine hypothetical scenarios in the literature of value of time. All decisions are made in their experiment without any actual time or monetary implications. Deacon and Sonstelie (1985) study waiting times and the decision process for gasoline purchases in 1980. They estimate that the value of time is approximately the after-tax wage rate. Arnott, de Palma, and Lindsey (1994) examine a single commuting route with heterogeneity of costs of being early and late to work. They predict that the time a commuter leaves for work depends on per-minute costs of travel time and arriving early to work. They also examine cases involving multiple starting times for work.

Two recent papers examine the value of time using commuting behavior in southern California. Both papers use commuters with the option to travel on either free or tolled lanes. Brownstone and Small (2005) find the typical estimates of the median value of time of actual morning commute decisions in many studies is in the $20-$40 range. Part of these estimates comes from Small, Winston, and Yan (2005). They find a revealed preference[49] median estimate for the value of time of $21.46 per hour. Commuters participating in this study are also asked to answer what they would do in hypothetical situations. These stated preference estimates for the values of time are considerably lower. Both papers above cite that a person's inability to properly estimate time savings may contribute to these differences.

In Segment 3 of Experiment 1, subjects are given explicit information that can help them to determine whether or not traveling the bridge in any round is worth the 6-point (or 12-cent) toll. The only uncertainty is that subjects do not know until the round is over how many people travel each route. The frequency of how often a particular number of subjects travel the bridge is listed in Table 10. Travel time on

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[49] Revealed preference estimates come from actual commuting behavior.

the bridge is lower in all rounds of Segment 3, except the one round when 11 travelers use the bridge, when the times are equal. The modal and median result occurs when five of 18 subjects travel the bridge, reducing their travel time at a rate of $12 per hour.

Table 10: Number of travelers on the bridge in Segment 3, Experiment 1.

Number of travelers on the bridge	Frequency, out of 200 rounds
1	1
2	10
3	18
4	23
5	50
6	42
7	31
8	16
9	7
10	1
11	1

Although a 95 percent confidence interval can be found for all 10 sessions collectively in Segment 3, Section 5.3.3 reveals that the group averages are significantly different from one another. Thus, I examine the confidence intervals of the number of bridge travelers by group, which are derived from the means, standard deviations, and a sample size of 20 rounds. These intervals are found in Table 11.

Two of the sessions include 4 bridge travelers in the confidence interval, six include 5, three include 6, and three include 7.

Table 11: Means and 95% confidence intervals for mean number of subjects on the bridge in Segment 3 of Experiment 1, by session

	Mean	Low end of 95% confidence interval	High end of 95% confidence interval
Session 1	6.35	5.56	7.14
Session 2	5.00	4.27	5.73
Session 3	6.50	5.86	7.14
Session 4	6.95	6.06	7.84
Session 5	5.00	4.26	5.74
Session 6	5.30	4.60	6.00[50]
Session 7	5.90	5.10	6.70
Session 8	5.00	4.39	5.61
Session 9	4.20	3.36	5.04
Session 10	4.70	3.94	5.46

Recall that Table 3 shows the per-hour cost to travel the bridge as a function of number of bridge travelers (C_T). From Table 11, there does not appear to be any group in which equilibrium is below four bridge travelers per round. Recall from Table 3 that if four people travel the bridge in Segment 3, theory predicts that each

[50] This value is rounded from about 5.997, which means that 6 is not quite in this 95% confidence interval.

person on the bridge has a benefit from the time saved that is equal to or greater than the toll cost. This implies at least four subjects in each group have values of time of $10.29 per hour or more. Table 11 also shows that there is not likely an equilibrium in which there are more than seven bridge travelers per round, implying that at most seven subjects in each group have values of time of $18 or more. Although these values of time are lower than those reported in Brownstone and Small (2005) and Small, Winston, and Yan (2005), they seem consistent with the earnings power of groups of college students that are mostly undergraduates.

6. Experimental Results and Analysis, Experiment 2

6.1. Data Summary and Comparison to Pure-strategy Equilibria

In Experiment 2, Sessions 2 and 5 follow format A, Sessions 1, 4, and 6 follow Format B, and Session 3 follows Format C. Thus, in the first segment of each session, Format C allows for no heterogeneity, Format B allows for low heterogeneity, and Format C allows for high heterogeneity. In the equilibrium derived from the theory in Section 3, the predicted equilibrium results in seven subjects on the bridge and eight on the highway.[51] However, in each of the formats, there exist possible equilibria such that six subjects are on the bridge (see Appendix B). These additional equilibria may explain why all sessions of Segment 1 average

[51] This is calculated by the fact that the seventh person on the bridge (with a 10 point per minute deduction) can reduce travel time by seven minutes, but must pay a 70-point toll. This trade off results in this subject being just as well off on either route.

6.88 bridge travelers per round, which is significantly lower than 7 and explain why Segment 2 of Session 1 has 22 rounds with exactly six subjects on the bridge.

Table 12 shows the average number of subjects on the bridge by session and segment, along with clusters of groups with the same format. In each session, the Segment 2 average is lower than in Segment 1. This may be due to learning between Segment 1 and Segment 2, especially if some subjects with relatively lower time costs can figure out that they cannot be better off on the bridge with more than seven subjects on the bridge. This may help to explain why some of the Segment 2 averages are significantly below 7.

6.2. Efficiency

Formats II and III achieve the minimum total travel time cost when seven subjects travel the bridge. In Format I of this experiment, the minimum total travel time cost occurs when six subjects travel the bridge, since a bigger negative externality is imposed on subjects with high values of time when additional subjects travel on the bridge. However, the next best outcome occurs when seven subjects travel the bridge. So in all formats, seven subjects on the bridge is at or close to the minimum total time cost.

6.3. Analysis

Three important aspects of this route choice experiment are analyzed below. First, when heterogeneity is introduced within the subject pool, most subjects have a

Table 12: Average number of bridge travelers per round in each segment, by session, Experiment 2.

	Segment 1	Segment 2
Session 1	6.84 (1.53)	6.44* (0.93)
Session 2	7.10 (1.31)	6.98 (1.12)
Session 3	7.06 (1.71)	6.74 (1.19)
Session 4	6.82 (1.53)	6.74* (0.66)
Session 5	7.20 (1.03)	7.06 (1.06)
Session 6	6.84 (1.81)	6.76 (1.20)
Sessions 1, 4, and 6	6.83 (1.62)	6.65* (0.96)
Sessions 2 and 5	7.15 (1.18)	7.02 (1.08)
All sessions	6.98 (1.50)	6.79* (1.05)

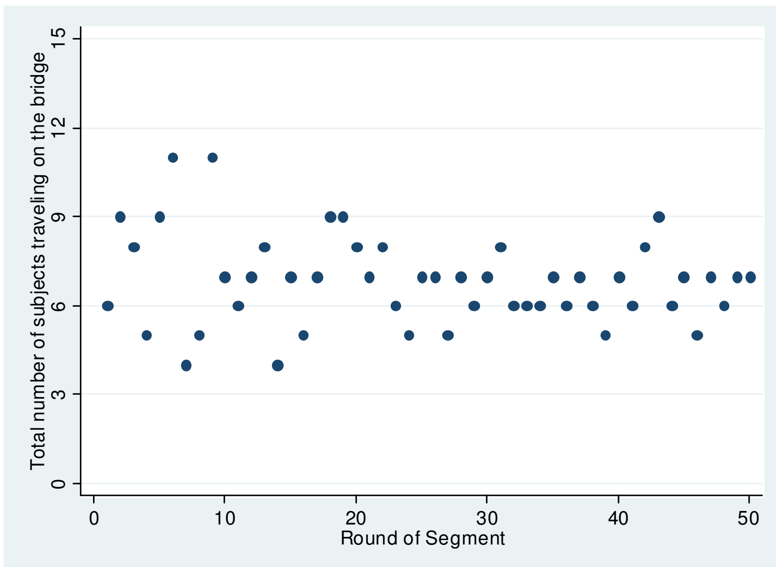
Standard deviations are in parentheses.

* denotes significantly different from 7.

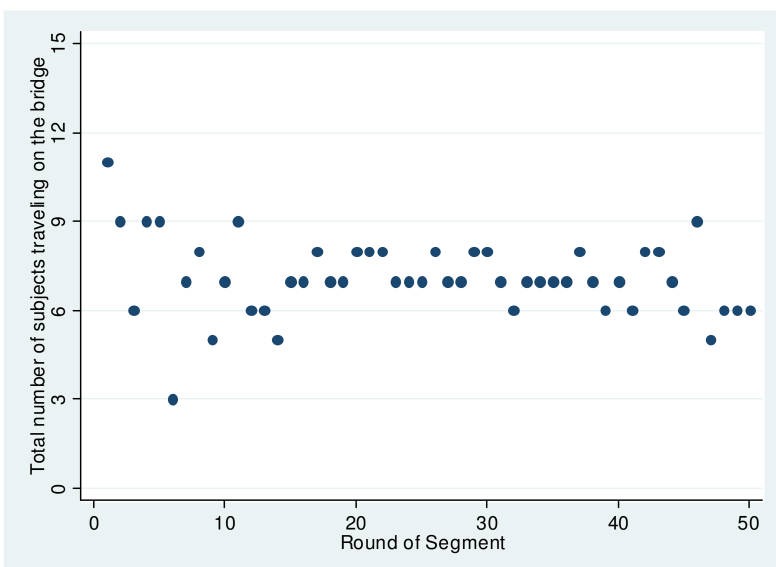
strict preference in equilibrium. This means that if subjects are profit maximizing (or minimizing their point deductions), then there should be less variability in the number of bridge travelers from round to round. I test to determine if there are

Figure 6: Round-by-round results of number of travelers on the bridge for each session and segment, Experiment 2

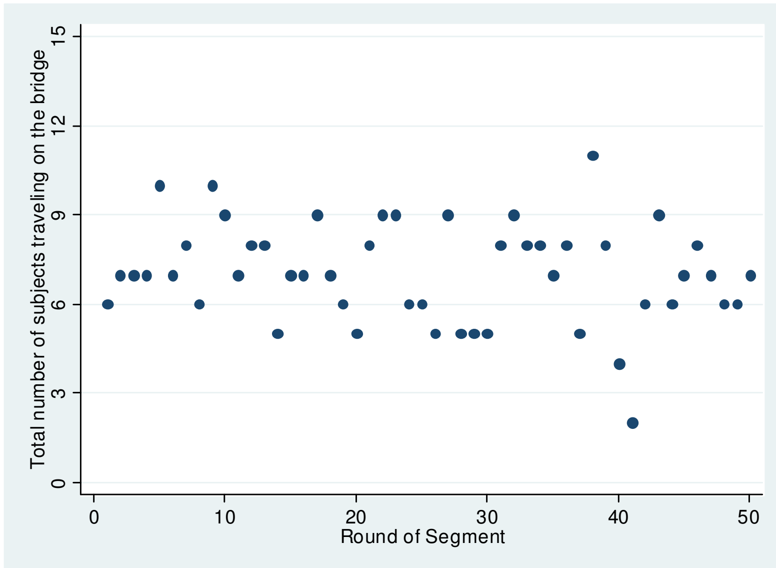
Session 1, Segment 1: Low heterogeneity



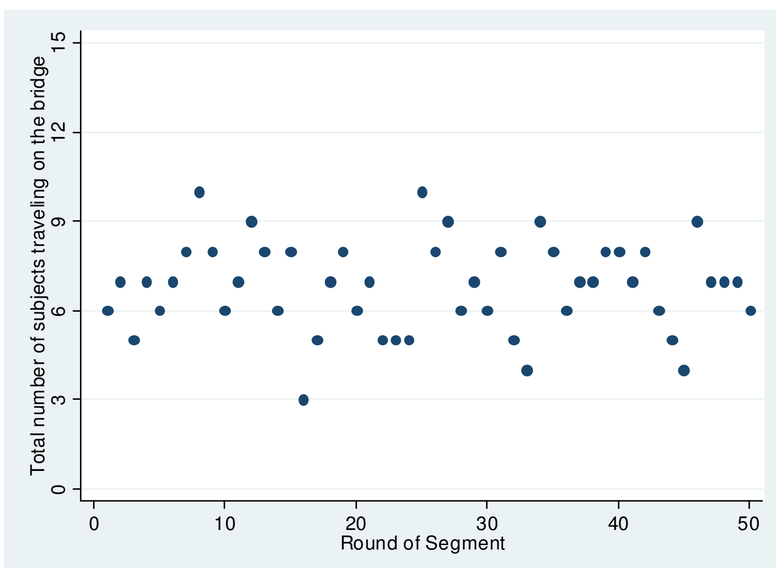
Session 2, Segment 1: High heterogeneity



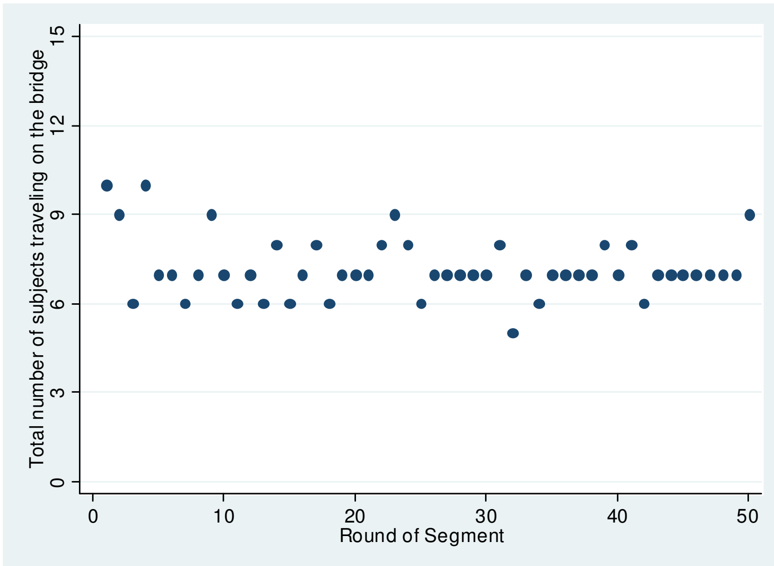
Session 3, Segment 1: No heterogeneity



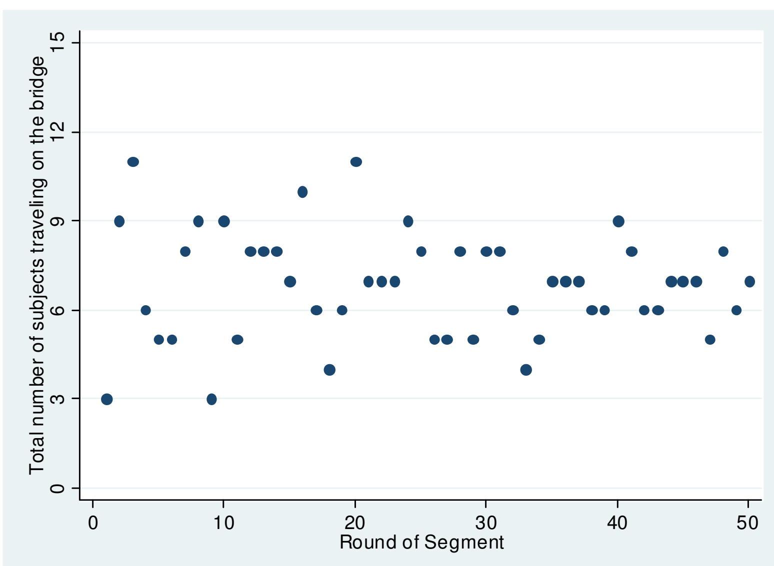
Session 4, Segment 1: Low heterogeneity



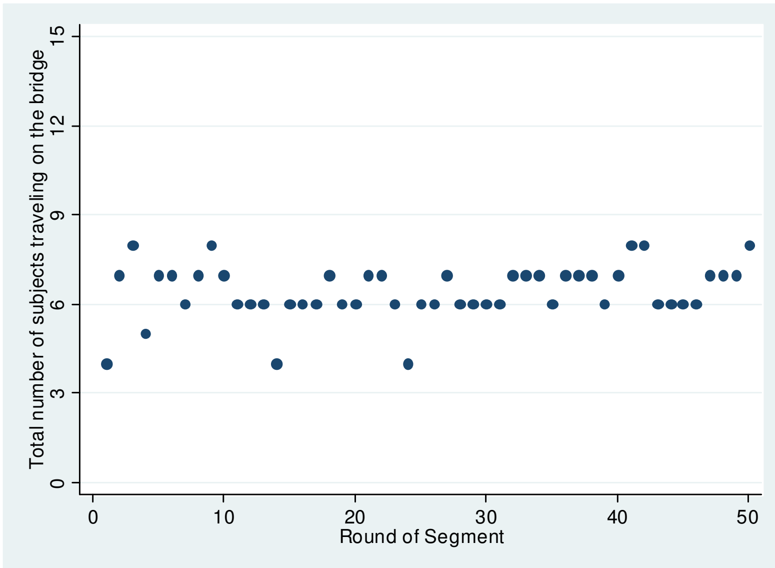
Session 5, Segment 1: High heterogeneity



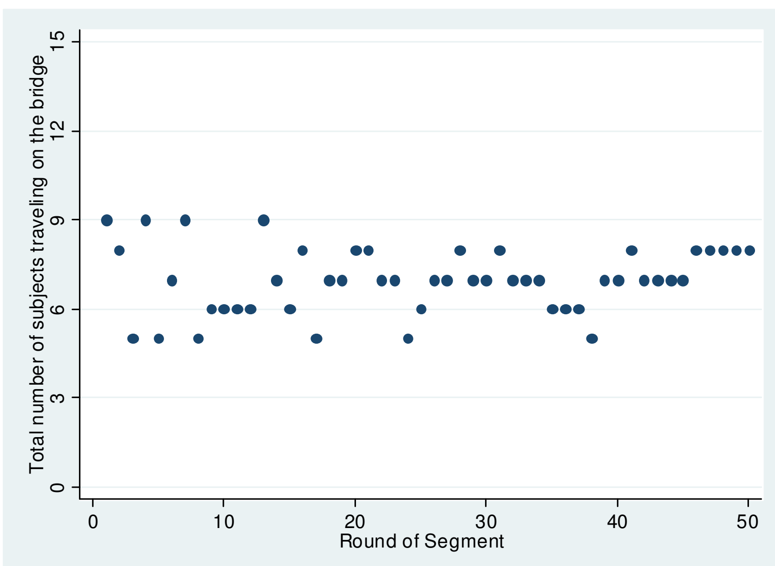
Session 6, Segment 1: Low heterogeneity



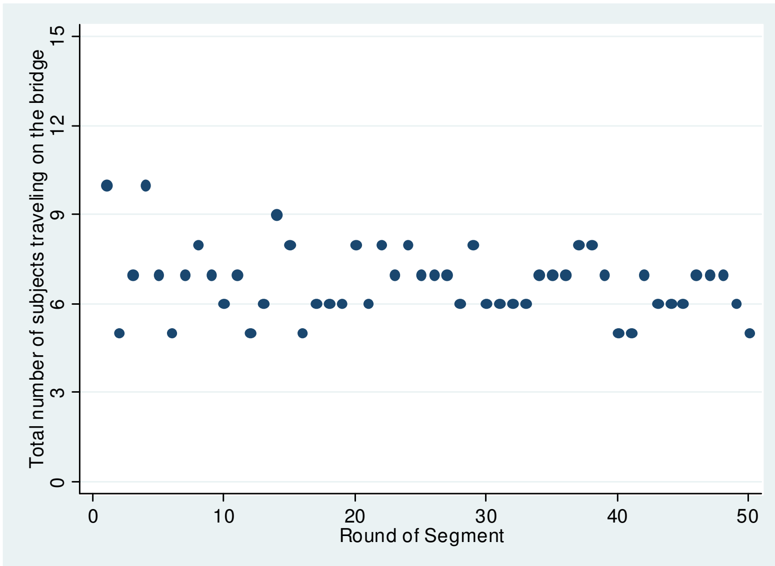
Session 1, Segment 2: High heterogeneity



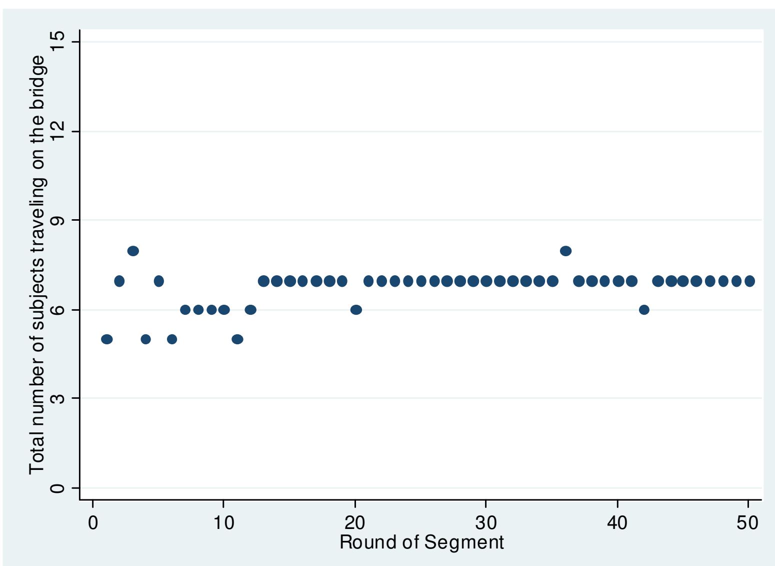
Session 2, Segment 2: Low heterogeneity



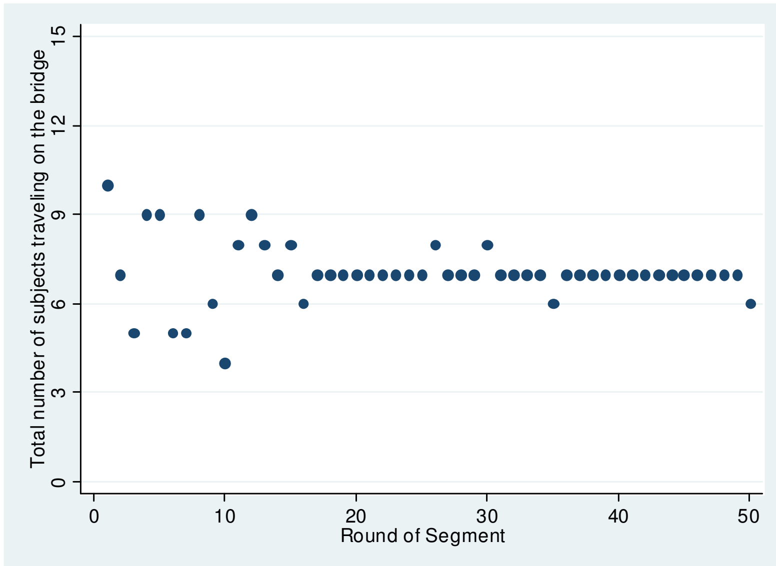
Session 3, Segment 2: No heterogeneity



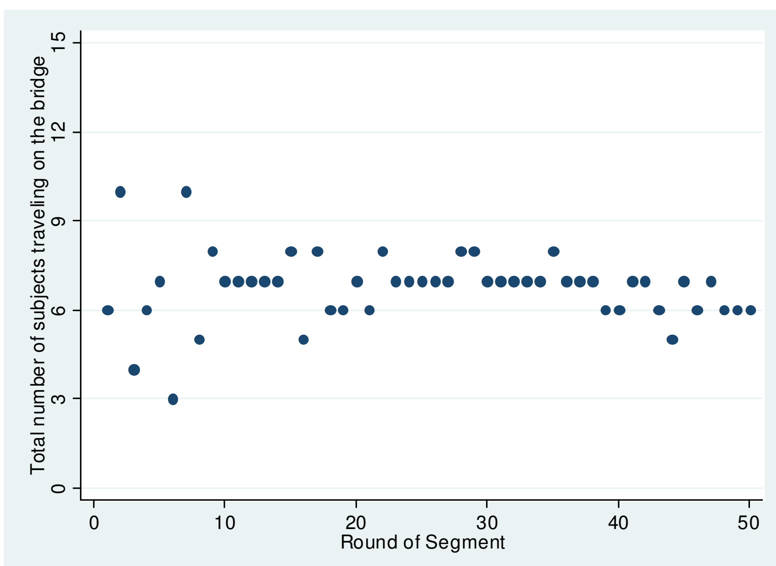
Session 4, Segment 2: High heterogeneity



Session 5, Segment 2: Low heterogeneity



Session 6, Segment 2: High heterogeneity



differences in variances when comparing two different levels of heterogeneity. Second, since each segment of each session has equilibrium of seven travelers on the bridge, I run various tests to determine if there are any differences of means. Third, I analyze how often subjects with point deductions not equal to 10 points per minute play the route that theory predicts they would play in equilibrium. After this analysis is completed, I address the lack of symmetric mixed-strategy equilibria in sessions with heterogeneity.

6.3.1. Differences in Variance as Heterogeneity Increases

As heterogeneity increases, the cost of deviating from equilibrium also increases for some people. This may result in subjects learning more quickly what is best for them when there is substantial subject heterogeneity, since the price paid for deviating increases. In this case, the variance in the number of bridge travelers in early rounds of sessions with high levels of heterogeneity will be lower. Figure 6 confirms this visually in Segment 1, as the sessions with high levels of heterogeneity have much less variation than the other sessions. This result is stronger if the final 25 rounds are examined in each session. In Segment 2, all sessions appear to have less variation in number of bridge travelers than in Segment 1.

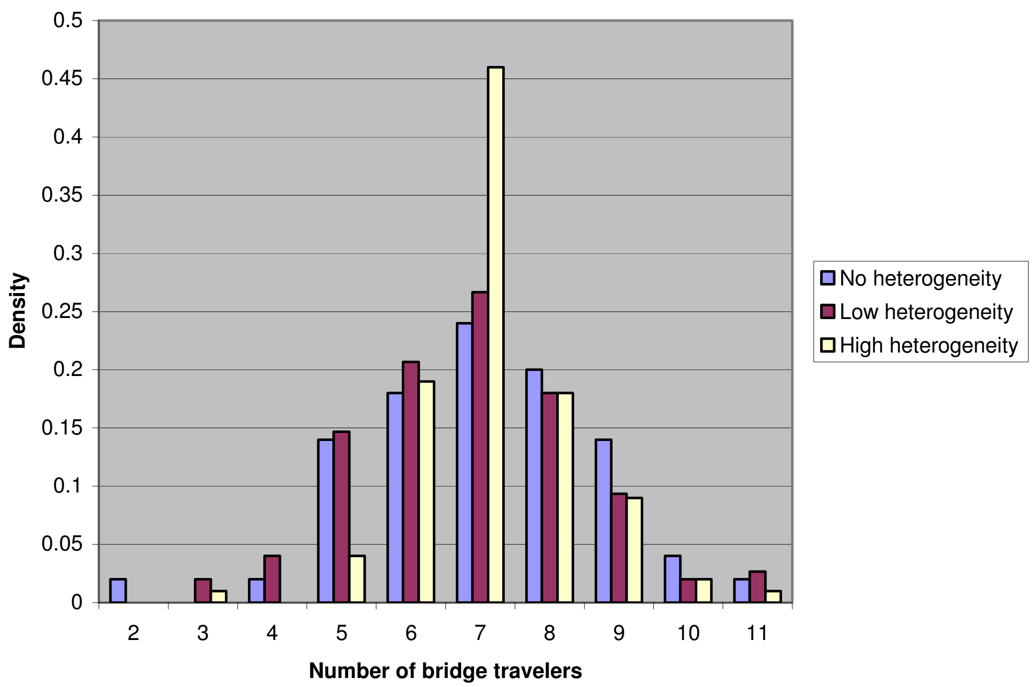
Table 13 confirms the above learning hypothesis for Segment 1 statistically, with high heterogeneity sessions having substantially less variance than the other sessions. These differences are significant at the 1 percent level. In contrast, the variance for low levels of heterogeneity is not statistically different from the variance with no heterogeneity. The variances of these two formats do not significantly differ. Variances decline between Segment 1 and Segment 2, with the only significant difference in Segment 2 being at the 10% level between the high heterogeneity and no heterogeneity formats. Apparently, the level of heterogeneity does not affect the variance of sessions once subjects are experienced with Segment 1.

Table 13: Tests of difference in variance by format

	Segment 1	Segment 2
High heterogeneity versus low heterogeneity	Ratio of variances: 1.899 Significant at the 1% level High heterogeneity has a lower variance	Ratio of variances: 1.262 Not significant
High heterogeneity versus no heterogeneity	Ratio of variances: 2.110 Significant at the 1% level High heterogeneity has a lower variance	Ratio of variances: 1.531 Significant at the 10% level High heterogeneity has a lower variance
Low heterogeneity versus no heterogeneity	Ratio of variances: 1.111 Not significant	Ratio of variances: 1.213 Not significant

One final way is used to show that learning occurs more quickly at high heterogeneity levels. Figure 7 shows that the sessions with high heterogeneity in Segment 1 have seven subjects on the bridge in over 45 percent of the rounds, while the other sessions only have this feature about one quarter of the rounds. The high heterogeneity sessions also have fewer rounds with less than six or more than eight on the bridge. By Segment 2, most of this difference is gone, although there are still a higher percentage of rounds in the high heterogeneity case with six to eight subjects on the bridge. This can be seen in Figure 8.

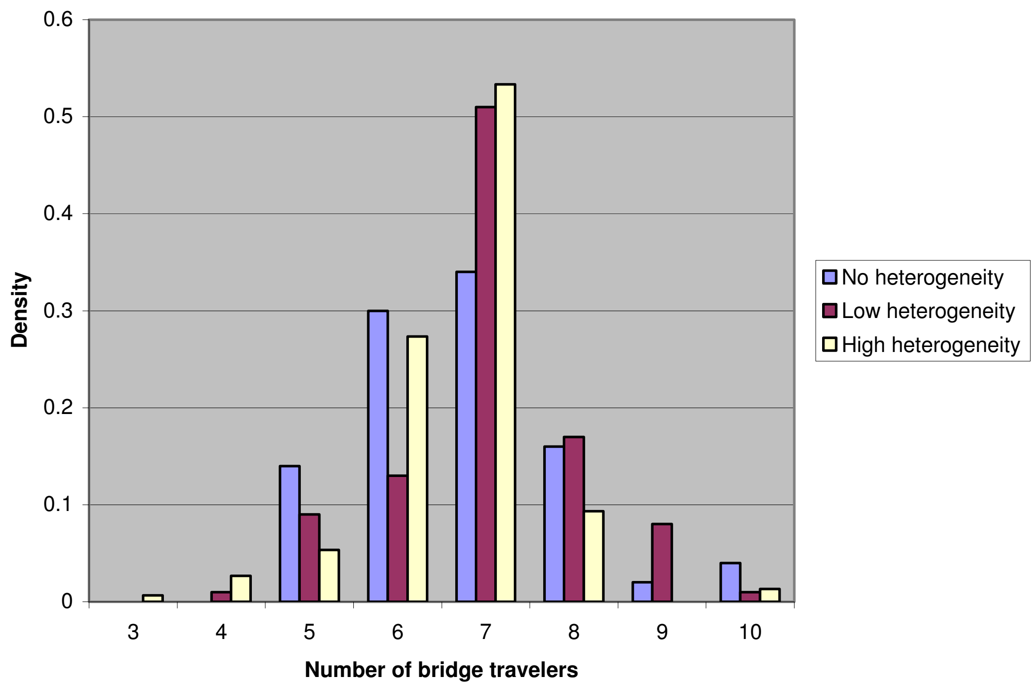
Figure 7: Distribution of number of bridge travelers in Segment 1 of Experiment 2



6.3.2. Variation in the Number of Bridge Travelers by Group

Since the predicted average number of travelers is seven in each segment of each session, it is useful to conduct tests to determine if any differences in the means exist. I conduct three types of tests here. In the first, I test if any segment averages

Figure 8: Distribution of number of bridge travelers in Segment 2 of Experiment 2



are different from 7, since this is the outcome in which all subjects with a 10-point-per-minute deduction are indifferent between routes.[52] This is followed by tests to determine if the segment averages differ from one another, either within a segment or over both segments. Finally, I conduct tests comparing clusters of sessions with the same level of heterogeneity.

[52] I also address the case of six drivers on the bridge per round to a lesser extent. This case seems less interesting due to the fact that drivers with a 10-point deduction per minute are not indifferent in this equilibrium.

The first test asks if any segment averages differ from seven bridge travelers per round. In most cases, I cannot rule out a null hypothesis of an average of seven drivers per round on the bridge.[53] Table 12 shows that no statistically significant differences occur in Segment 1. However, the averages in Segment 2 of Sessions 1 and 4 differ from 7. The average for all of the high heterogeneity sessions in Segment 2, Sessions 1, 4, and 6, also differ from 7, as does the average for all six sessions. In all of these cases, the averages are lower than 7, which suggest that some sessions may be affected by the equilibrium possibility of 6 subjects on the bridge in Segment 2. For example, in the case of Session 1, the group of subjects alternates between six and seven subjects on the bridge throughout most of the segment. However, in Session 4, the significant difference is due to most of the rounds being at 7 bridge travelers (leading to low variance) combined with more rounds below 7 than above it.

The next test asks whether or not segment averages differ from one another. This is done two different ways. First, I test to determine if there are any differences in means of all sessions within the same segment. There are no significant differences in Segment 1, but the differences are marginally significant in Segment 2, with a p-value of 0.057. The second method checks to see if there are any differences of means by segment within the same session. Although the average

[53] I also conduct the same tests with the null hypothesis of an average of six drivers on the bridge per round, since this is also a possible pure-strategy equilibrium. In all cases, the null hypothesis is rejected.

number of travelers in each session decreases going from Segment 1 to Segment 2, none of these differences is significant.

Table 14: Tests of differences in means by format

	Segment 1	Segment 2
High heterogeneity versus low heterogeneity	|t| = 1.79 Significant at the 10% level High heterogeneity has a higher average	|t| = 2.79 Significant at the 1% level Low heterogeneity has a higher average
High heterogeneity versus no heterogeneity	|t| = 0.34 Not significant	|t| = 0.50 Not significant
Low heterogeneity versus no heterogeneity	|t| = 0.82 Not significant	|t| = 1.40 Not significant

The final test asks whether or not there are differences by format. More specifically, I look at two of the three different formats at a time — with each format having a different amount of heterogeneity[54] — and conduct a t-test to determine if there are more travelers in one of the two formats. These tests are displayed in Table 14. The only cases with significantly different averages occur in the comparison between high and low levels of heterogeneity. However, the higher average is for Sessions 2 and 5 in both cases. Thus, in Segment 1, the sessions with high heterogeneity have more travelers on the bridge per round. In Session 2, the low-

[54] Recall that Format I is called "high heterogeneity," Format II is called "low heterogeneity," and Format III is called "no heterogeneity."

heterogeneity sessions have more bridge travelers per round. This means that the differences are based on the groups of subjects or experimental design and not the amount of heterogeneity.

6.3.3. Frequency of Equilibrium Play by Subjects

In this experiment, most types of subjects can only be on one route in equilibrium. From Appendix B, any subject with a point deduction of 12 or more per

minute of travel time must be on the bridge in equilibrium, while those with 9 or fewer points deducted per minute of travel time must be on the highway in equilibrium. However, following the prediction made by Section 3.3, those with 11 points deducted per minute will also travel on the bridge in equilibrium, as will exactly one out of three with 10 points deducted per minute when heterogeneity of point deductions is present.

Table 15 shows the average number of trips on the bridge per subject, by per-minute point deduction and segment. Excluding the control group, the average number of trips is positively correlated with the number of points deducted per minute. However, subjects appear to act more closely to equilibrium prediction in Segment 2 than in Segment 1 in the low heterogeneity format. This may help to explain the lower variance of number of bridge travelers in Segment 2 of this format.

Although most subjects follow the equilibrium prediction in most rounds when there is a strict preference, there are some that do not. For instance, three of the nine subjects with 9 points deducted in Segment 1 travel on the bridge more than half the time during these 50 rounds. Another example is of a subject with a 4-point deduction per minute in Segment 2. This person travels on the bridge 38 of 50 rounds in Segment 2. Another subject in Segment 2, with a 9-point deduction per minute, travels the bridge 49 of the 50 rounds. Finally, there are subjects with 12- and 13-point deductions per minute in Segment 2 travel the bridge five and 11 times, respectively.

Table 15: Average number of trips per subject on the bridge, by point deduction and segment, with standard deviations in parentheses

	Segment 1	Segment 2
4 points per minute	3.83	6.00
	(4.02)	(12.05)
7 points per minute	4.33	2.11
	(3.01)	(1.69)
8 points per minute	12.44	6.50
	(9.46)	(6.02)
9 points per minute	19.44	13.83
	(13.34)	(17.59)
10 points per minute	18.67	18.80
(excluding format III)	(12.19)	(19.69)
11 points per minute	25.44	37.33
	(20.56)	(17.49)
12 points per minute	41.44	36.83
	(5.43)	(15.92)
13 points per minute	44.83	41.78
	(2.71)	(12.11)
16 points per minute	42.17	44.56
	(6.88)	(5.90)
10 points per minute	23.53	22.47
(format III only)	(12.11)	(21.08)

Two subjects not acting in a way that is consistent with equilibrium can make the aggregate appear to look like it is in equilibrium. For example, in Session 2, one person with 9 points deducted per minute travels the highway only once, while a person with 12 points deducted per minute almost never travels the bridge. Although most of the later rounds of this segment are at or around seven bridge travelers per round, these two people should reverse roles in equilibrium. However, by essentially switching roles, the other 13 people can still reach their best outcome with seven subjects on the bridge in total.

Another example clearly shows that a subject is acting out of equilibrium. Any subject with a 4-point deduction per minute can never be better off on the bridge, since the lowest possible point deduction on the bridge occurs if she is the only traveler on this route. In this case, the time cost is 48 points, and the toll is 70 points, leading to a total deduction of 118 points. If she travels the highway, her travel time is 25 minutes with no toll, resulting in a point deduction of 100 points.

Despite the fact that any person with a 4-point deduction per minute should always travel the highway in this experimental set-up, there is one person in Segment 2 of this type that travels on the bridge in each of the first 38 rounds.[55] This person, a subject in Session 6, likely caused each of the subjects with 10 points deducted per minute to travel the bridge infrequently in Segment 2. If the irrational play described above is assumed, then the other 14 subjects can reach equilibrium

[55] Not surprisingly, this is the subject that lost all points in the 91st round. This subject does travel on the highway in the final 12 rounds of the experiment, however.

when all other subjects with 10 points or fewer deducted per minute travel the highway. In this session, the three people with this profile travel on the bridge 1, 4, and 19 times each. This compares to the 22.25 trips per subject for counterparts in the segments of other sessions with high heterogeneity. Thus, in Segment 2 of Session 6, subjects with 10 points deducted per minute of travel time seem to become averse to traveling the bridge due to the decision by the person traveling the bridge most of the rounds who has 4 points deducted per minute. This results in a segment that looks like it approaches equilibrium in the aggregate, except for the final 12 rounds in which the most frequent outcome in this part is with six subjects on the bridge.

6.3.4. Symmetric Mixed-Strategy Nash Equilibria

In Experiment 2, there is no symmetric mixed-strategy Nash equilibrium in Formats I and II, since some players have strict preferences in equilibrium. In the control, Format III, a symmetric mixed-strategy Nash equilibrium can be calculated similarly to those in Experiment 1. Using the same method as in Appendix A, the mixed-strategy Nash equilibrium consists of each subject on the bridge with probability 3/7, or 0.429.

6.3.5. Summary of Analysis

In most of the individual segments of this experiment, the average number of bridge travelers is consistent with the equilibrium of seven bridge travelers per round. Although possible equilibria exist with six bridge travelers per round on the bridge, none of the averages is consistent with this as the outcome. However, the existence of possible equilibria with six subjects on the bridge may contribute to some of the averages being different from both 6 and 7.

Once heterogeneity is introduced into any format of experiment, most subjects can only play a pure strategy in equilibrium. This leads to two important points. First, this is the likely cause of lower variation in Segment 1 when there are high levels of heterogeneity, since subjects that deviate from equilibrium pay a high price to do so in this situation. Second, some subjects do not play in a profit-maximizing way when heterogeneity is present. When one person behaves in such a way, there is evidence that others adjust their decisions to attempt to reach their best payout. In another case, two people act in such a way. Here, they essentially trade their route choice decisions, and so the other 13 people can still behave in a way that is consistent with a predicted equilibrium outcome.

7. Conclusion

Although billions of dollars are spent each year on transportation, Americans seem to lose more time due to congestion each year. In many urban areas, the problems are so bad that "building our way out of congestion" is now prohibitively expensive. One of the few viable solutions to this problem involves applying tolls during congested periods on roads and highways. The cost of technology needed to implement such tolls should now be low enough to ensure that generated toll revenue is sufficient to make substantial Pareto improvements possible.

While theory certainly supports the use of tolls as a mechanism for reducing congestion, there is only limited empirical and experimental evidence examining the functioning of such plans. Perhaps more importantly, until this paper there has been no experimental evidence that allows for commuter heterogeneity. Consistent with standard theory that assumes homogeneity in time costs across all commuters, on average, the Segment 1 results of Experiment 1 show that inefficient levels of congestion occur when no tolls are imposed. When tolls are imposed in the same homogeneous time cost framework in Segment 2 of Experiment 1, the results match the theoretical prediction that fewer subjects choose the congested route. The heterogeneity in the value of time in Segment 3 of Experiment 1 leads to equilibria that are session-specific based on the distribution of values of time within the session. Experiment 2 verifies that people with high values of time are more likely to pay a toll to reduce commuting times than others with low values of time. So while congestion experiments with homogeneous subjects are able to examine some aspects of behavior, subject heterogeneity is needed to examine how individuals with different values of time trade off time against money. This is crucially important because people in actual traffic environments must make this decision every day.

Further, heterogeneity also helps explain why similar traffic networks could have different commuting patterns when they serve different populations.[56]

Although the results reported in this paper, and previous experimental congestion papers, answer some important questions about congestion behavior, further research is necessary to address some nagging problems in congestion experiments. First, the existing experimental results do not match perfectly with congestion theory. In Experiment 1, a round in disequilibrium often follows a round in equilibrium in many experiments. In Experiment 2, some subjects do not figure out the route that minimizes their total point deduction, but there is substantially less fluctuation in number of travelers on each route from one round to the next than in Experiment 1. Second, experiments on congestion have primarily focused on automobile congestion. In recent years, congestion has surfaced more frequently in American electricity markets. Unless forced power outages can be imposed when demand exceeds available supply, entire grids are vulnerable to overloads. While externalities from traffic congestion are usually gradual, those from electricity grid congestion arise quite quickly when demand reaches capacity.

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[56] The use of tolls is just one of many variables that could affect different populations of travelers in different ways. Others include the length of the commuting period, the elasticity of demand for travel, employer flexibility of starting and ending of work times, and land use patterns.

II. A Comparison of Individual and Group Behavior of a Route Choice Experiment, Examining Pure and Mixed Strategies

1. Introduction

Economic theories typically assume that agents act rationally. Economists are well aware that real people often do not act as these theories predict. J.R. Hicks (1956), in his critique of the theory of revealed preference, wrote that economics is not concerned with predicting the choices of individuals, but rather to predict the average of their individual choices. He wrote " …the preference hypothesis only acquires *prima facie* plausibility when it is applied to a statistical average. To assume that the representative consumer acts like an ideal consumer is a hypothesis worth testing; to assume that an actual person, the Mr. Brown or Mr. Jones who lives round the corner, does in fact act in such a way does not deserve a moment's consideration" (Hicks 1956, p. 55). In other words, economists should be able to explain market demand, but should not be expected to foretell the demand of an individual. So if all economists care about is market outcomes, it does not matter if individuals play as predicted, as long as the aggregate outcome is consistent with the predicted outcome.

Müller and Schotter (2007) examine cases in which behavior on the individual level does not conform to theory, although the aggregated behavior is consistent with theory. They test a theory related to the awarding of prizes in a contest in which all contestants pay in the form of effort.[57] They find that when the population is examined as a whole, theory predicts quite well the effort levels given ability, which looks like a continuous relationship. However, on the individual level, subjects often behave in a more discrete manner, either providing high effort when ability is high or very low effort when ability is low.

Rapoport *et al* (2004) report in a complicated queuing experiment that when models are compared to individual behavior, the predictions do not match well with the results. Their experiment involves choices of arrival times at a (fictitious) auto inspection station. While considerably more complicated, the experiment has many similarities to the one reported here. Subjects receive an initial allocation of points (called francs) with a known rate of conversion into dollars. Unlike the results in this paper, in which individuals can only be penalized for time spent, their experiment allows subjects to be rewarded for completion of a job as well as being penalized for time spent. Like the game described in the next section, theirs has a unique mixed-strategy equilibrium. The particularly interesting similarity is that they find, as do I, that "[t]here is clearly no support for equilibrium play on the individual level" (p. 83). However, "…the mixed-strategy equilibrium solution accounts for the aggregate results remarkably well" (p. 77).

One other paper that examines individual and aggregate behavior is by Levine and Palfrey (2007). They model the likelihood of voting as a function of

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[57] Subjects do not exert actual effort, but enter an effort, which has a monetized time cost attached to it.

cost. While theory matches well with aggregate behavior, they also find that individual behavior is inconsistent with Nash equilbrium.

Walker and Wooders (2001) look at how experience affects the ability to use strategies that economists would predict. They test the minimax hypothesis on top tennis professionals, along with an experiment not related to tennis. These experimental subjects are probably less experienced at using mixed-strategy Nash equilibria to their advantage. What they find is that the tennis players mix up their serves in a way that is consistent with the minimax hypothesis, but they change their service direction more often than what this hypothesis predicts. The experimental data shows that both mixing proportions and frequency of switching are inconsistent with the minimax hypothesis.

What I present here is another case in which various models of individual choice do well in predicting the aggregate outcome, but are not supported by individual behavior. The example used below is an experiment based on the Pigou-Knight-Downs (PKD) Paradox.[58] This is a curiosity from transport economics. Because it is so simple to understand, and since a large percentage of the population drives, it lends itself well to experimental verification.[59]

A condensed version of the paradox is simple to explain with an example. There are two locations, I and II, separated by some distance. Their only connection

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[58] So labeled by Richard Arnott and Kenneth Small (1994).

[59]Other experiments that have dealt with commuter route choice include Selten *et al* (2007) and Gabuthy, Neveu, and Denant-Boemont (2006). In both papers, oscillations around any pure strategy equilibrium are typically seen.

is a highway, H, on which it takes a fixed amount of time, *T*, to travel between the two places, irrespective of the number of commuters (H is not congestible). A second, but congestible link, B (for Bridge) is built. Because, at low density, B takes less time than H, commuters will switch from H to B. If there are a sufficiently large number of commuters, they will continue to switch until the travel time on B rises to *T*. The paradox, of course, is that travel time on B equals that of H, and total travel time is not reduced by the construction of B.

The paradox is appealing because it is so simple to compute. But the paradox is not a certainty; it is really more a prediction of what will be observed in the situation described. It seems appealing that if people see no advantage to one road or another, they will decide to maintain the choice made already. In that case, once equilibrium is achieved, it will maintain itself forever. There is in this view an implicit dynamic assumption, namely, the maintenance of choice when there appears to be no advantage to a change. There is, however, no model of how the equilibrium state is achieved.

The PKD equilibrium is an equilibrium in pure strategies. However, subjects may also play mixed strategies. Each person's mixed strategy could be the same during each commute between locations I and II, or it could depend on one or more previous commutes. In this paper, I examine three mixed strategies that are independent of previous commuting history, along with one that depends on the commute immediately before.

This paper compares people's behavior of a route choice experiment in the PKD framework, both at the individual and the aggregate levels. Since the framework is simpler than some of the other experiments described above, various theories can easily be compared to the experimental outcomes. These experimental sessions, consisting of mostly undergraduate students at the University of California, Santa Barbara, are offered monetary payoffs that decline based on fictional travel time in the experiment. What I find is that heterogeneity of subjects' reactions affects the aggregate outcomes in such a way that groups of subjects appear to be acting similarly to various mixed strategies if only the aggregate results are looked at.

2. Experimental Design

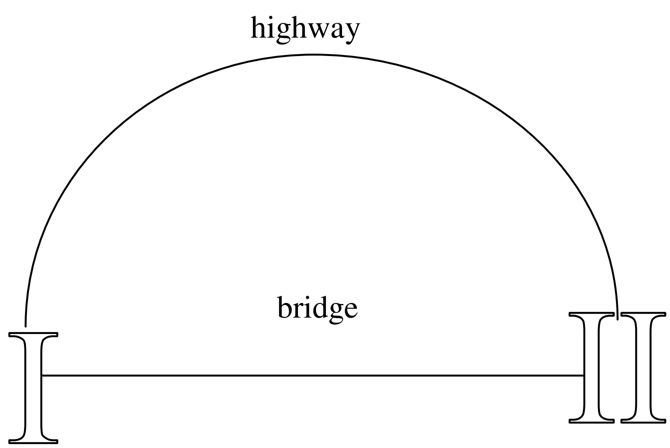
Eighteen subjects[60] must travel from point I to point II using either a congested bridge or an uncongested highway in each round (see Figure 9).[61] The highway guarantees a travel time of $t_H = 20$ minutes. In contrast, while the bridge is uncongested for the first traveler, and hence has a travel time of 10 minutes, each additional driver on the bridge adds one minute to every bridge user's travel time.[62] In other words, if there are T subjects traveling the bridge in any round, the travel

[60] Only 17 subjects participate in one group. This group is ignored in the analysis of this paper.

[61] The experiment is programmed and conducted with the software z-Tree (Fischbacher forthcoming).

[62] Note that if all subjects travel the bridge, each subject requires 27 minutes in travel time.

Figure 9: The travel network used in the experiment



time for each person is $t_B = (9 + T)$ minutes. Each subject may stay on the same route or change from round to round, but no one is permitted to change their choice within a round once their decision has been made. At the end of each round, subjects receive information as to how many people travel on the bridge for that round.

Each group participates in three segments consisting of 20 rounds (or repetitions) each, and each subject begins with 8500 points and a $5 show-up fee. Points are deducted for travel time in the first two segments, but not for the third segment. Instead of paying a monetary equivalent for time in Segment 3, subjects are told that more travel time within this segment results in an increased physical waiting time before receiving their payment at the end of the experiment. Finally, tolls are charged (in the form of point deductions) on the bridge in the final two segments of the experiment. After the experiment is finished, the remaining points are converted at a rate of 50 points per $1.

This paper looks at subject behavior in Segment 1 of this experiment. However, brief descriptions of Segments 2 and 3 are also included for completeness and convenience. Full details of the experiment can be seen in Chapter I.

Segment 1: The No-Toll Case With Homogeneous Time Costs

Subjects are told that each minute of travel time leads to a 10-point deduction, but no tolls are charged. Assuming that all subjects are profit maximizers, they attempt to choose the route that minimizes their point deduction in every round. This, of course, means that their route choice depends on their expectations about what the other 17 subjects will do in any particular round.

Segment 2: The Toll Case With Homogeneous Time Costs

Subjects continue to pay a 10-point deduction per minute of travel time, but now there is a 60-point-per-round toll charge. As shown in Chapter I, a 60-point toll translates to the equivalent of six minutes of travel time cost. This means that a 14-minute commute on the bridge is now equivalent (in total point deductions per round) to a 20-minute commute on the highway. This framework leads to a pure strategy Nash equilibrium that minimizes total travel time and is efficient.

Segment 3: The Actual Waiting Time Case

In this part of the experiment, subjects can literally make trade-offs of money for waiting time. A subject only pays a 6-point toll charge to use the bridge in Segment 3, but no longer faces a point deduction for travel minutes in the experiment. Instead of a point deduction for travel time, a subject's sum of travel minutes for the 20 rounds in this segment is converted into waiting time at the end of the segment.

3. Models of Modal Choice, Changing Routes, and Aggregate Traffic

There are many models that can be used to model individual route choice behavior. I first describe the prediction made by the Pigou-Knight-Downs (PKD) paradox, which leads to an equilibrium in pure strategies. This is followed by four mixed strategies, one of which depends on the commute in the previous round. The section ends with an introduction to Yule coefficients, which examines the heterogeneity of the subject pool in reacting to relatively good and bad outcomes.

3.1. Pigou-Knight-Downs (PKD)

From the Pigou-Knight-Downs paradox described in the introduction, the predicted equilibrium occurs when travel times are the same on both routes (see Arnott and Small 1994). For this outcome to occur, the congestion on B must be such that any additional travelers on B will lead to B having a higher travel time than H, and any fewer travelers on B will lead to B having a lower travel time than H. The PKD framework predicts an equilibrium with 20 minutes of travel time on both routes. This occurs when $T = 11$. This is also a Nash equilibrium in pure strategies, since any person that deviates from this equilibrium is not better off.[63]

[63] Assuming that nobody else switches routes, anybody switching from B to H will be just as well off, while anybody switching from H to B will be made worse off, due to the increased congestion that this person causes.

3.2. Coin-Flip (CF)

The Coin-Flip (CF) prediction, as is the PKD prediction, is silent about the dynamic path to equilibrium, but it imagines possible individual responses when there are no time differences between the two routes. The CF model assumes that once the travel times are equal in PKD equilibrium, each person will randomly choose one of the two routes with equal probability. The CF theory thus involves individual choice behavior in commutes immediately following a commute in PKD equilibrium. In these cases, the predicted outcome is a probability distribution, rather than a deterministic prediction. This probability distribution is displayed in Table 16. While this is a small variant on the PKD model, it leads to considerably different predictions about road use. With PKD, once the equilibrium is achieved that will be the distribution of the highway and bridge users forever. Thus, the PKD equilibrium is an absorbing state. That is not the case for the CF model. Every feasible distribution of use (from all on the highway to all on the bridge) is possible. The CF model does not predict a certain value of bridge use, but rather a probability distribution.

3.3. Mixed-Strategy Nash Equilibrium (MS)

Although it is possible that drivers randomize after a round in PKD equilibrium, there is only one symmetric mixed-strategy Nash Equilibrium that is possible in this route network. Any person can have a guaranteed travel time on H,

Table 16: **Experimental distribution and predictions from other models**

Number of Bridge Commuters	Experimental Distribution	PKD	Logit Steady State	MS Distribution	CF Distribution	Naïve Decision Distribution
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0.001	0
3	0	0	0	0	0.003	0
4	0.006	0	0.001	0.001	0.012	0.001
5	0.011	0	0.003	0.006	0.033	0.003
6	0.011	0	0.010	0.018	0.071	0.012
7	0.039	0	0.029	0.045	0.121	0.031
8	0.072	0	0.068	0.088	0.167	0.067
9	0.111	0	0.123	0.140	0.185	0.118
10	0.166	0	0.177	0.179	0.167	0.166
11	0.194	1	0.201	0.186	0.121	0.190
12	0.167	0	0.177	0.155	0.071	0.174
13	0.106	0	0.120	0.102	0.033	0.126
14	0.067	0	0.062	0.052	0.012	0.071
15	0.033	0	0.023	0.020	0.003	0.030
16	0.006	0	0.006	0.005	0.001	0.009
17	0.006	0	0.001	0.001	0	0.002
18	0.006	0	0	0	0	0
Mean number of commuters	10.92	11.00	10.94	10.59	9.00	11.00
Variance	4.74	0.00	3.79	4.36	4.50	4.28
Sum of Square Differences	0	0.7406	0.0010	0.0020	0.0492	0.0008

while the expected travel time on B depends on the probabilities of other travelers of traveling on B. Let the travel time on B take the form

(13) $t_B = (X + YT)$ minutes,

where T denotes the number of travelers on B, $X + Y$ denotes the travel time if one person travels on B, and Y denotes the additional travel time on the bridge to all drivers due to the additional congestion caused by one more driver on the bridge. Let N denote the number of total drivers traveling from point I to II. In order for the expected travel times to be equal, the following condition must hold, where an individual driver assumes that p_{Nash} represents the probability of each of the other $T - 1$ drivers of traveling B:

(14) $X + Y \times (1 + (T - 1) \times p_{\text{Nash}}) = t_H$

This yields p_{Nash} necessary to produce the symmetric mixed strategy Nash equilibrium of

(15) $$p_{\text{Nash}} = \frac{t_H - X - Y}{Y \times (T - 1)}.$$

Since $t_H = 20$, $X = 9$, $Y = 1$, and $T = 18$, then $p_{\text{Nash}} = 10/17$, or about 0.588. Again, the predicted outcome is a probability distribution, displayed in Table 16.

3.4. Naïve

Since drivers may not be sophisticated enough to figure the MS equilibrium, a more Naïve model is described here. Here, drivers randomize their choices in the same way as to get an expected number of total drivers on the bridge to equal the PKD prediction. Using the given parameters in the MS subsection above, the probability of all drivers needed to fulfil this condition is

(16) $$p_{\text{Naïve}} = \frac{t_H - X}{Y \times T}.$$

In Segment 1 of this experiment, $p_{\text{Naïve}} = 11/18$, or about 0.611.

3.5. Logit Stochastic Choice Model

In the models described above, strategies in each commuting period are typically silent about how the history of commuting patterns plays a role (as in the PKD model) or strategies are independent of history of commuting patterns (as in the MS and Naïve models). A model that lends itself well to the analysis of the experimental data is what I call the Individual Response to Current State (IRtCS). This simple idea states that individuals assess the changes of tomorrow's traffic by observing today's. On the basis of that assessment (along with their choice of today's route), they choose the option that they believe will minimize travel time tomorrow. The model does not specify how individuals assess the chances of future states, it simply allows for their assessments to be different. The IRtCS, when modeled as a Logit Stochastic Choice Model (Logit) similar to McKelvey and Palfrey (1995), nests PKD, MS, Naïve and CF, which allows to test the models within this framework.

The Logit model of route choice allows for the possibility that the probability law that governs the choice for those taking the highway may be different from that of the bridge takers. In this model, the probability that an individual on the highway today will choose to switch routes tomorrow is related to the number of bridge travelers today:

(17) $$P_{H_{t+1}|H_t}(\Delta_t) = \frac{1}{1+e^{-\alpha_H - \beta_H \Delta_t}},$$

where $\Delta_t = N_B - 11$ is the difference between the number of commuters on the bridge today (time period *t*) and the PKD equilibrium. Similarly, the bridge-taker probability is

(18) $$P_{B_{t+1}|B_t}(\Delta_t) = \frac{1}{1+e^{-\alpha_B - \beta_B \Delta_t}}.$$

With this specification of choice probabilities, I expect if there is a rational relation between current traffic and individual choice, that the β coefficient is positive for H and negative for B. To understand this rationale, consider the derivative of either equation with respect to Δ. If behavior is consistent with our expectation it should be positive for H, as the current bridge traffic results in the probability of remaining with the highway increasing. For the similar reason, the derivative for the bridge should be negative. The derivative is

(19) $$\frac{dP_{k_{t+1}|k_t}}{d\Delta} BH = -\beta \frac{e^{-\alpha_k - \beta_k \Delta_t}}{(1+e^{-\alpha_k - \beta_k \Delta_t})^2} \overset{<}{\underset{>}{=}} 0 \text{ if } \beta \overset{<}{\underset{>}{=}} 0, k \in (B, H).$$

3.6. Yule Coefficients and Changing Routes

People respond to success and failure in different ways. It is therefore useful to determine how people react in these situations. In the first two segments, it is clear to subjects when they are performing better than those travelling the other route. In any round of the first two segments, a subject may decide to travel the highway and guarantee a 20-minute travel time, which leads to a 200-point deduction. The Yule coefficient examines how subjects react to good and bad outcomes in a particular round of Segment 1. A good outcome is defined as traveling on a route with a strictly lower point deduction than the route not traveled in a round, while a bad outcome is the opposite. In a neutral outcome, all subjects have 200 points deducted.

Using the terminology from Selten *et al* (2007), a direct response is one where a subject who has a bad outcome one round changes her route the following round, or has a good outcome followed by traveling the same route. A contrarian responder is one who has a good outcome and changes her route the following round, or has a bad outcome and follows by traveling the same route. Let c_- represent the number of times the route is changed after a bad outcome by a subject in Segment 1, c_+ represent the number of times the route is changed after a good outcome, s_- represent the number of times the route stays the same after a bad outcome, and s_+ represent the number of times the route stays the same after a good outcome. (See Table 17 for a 2-by-2 grid.) Then the subject's Yule coefficient (*Q*) can be calculated as follows:[64]

(20) $$Q = \frac{c_{-} \times s_{+} - c_{+} \times s_{-}}{c_{-} \times s_{+} + c_{+} \times s_{-}}.$$

The most extreme Yule coefficients possible are 1, in which a subject tends to be a direct responder, and –1, where a subject tends to be a contrarian responder.

Table 17: Route choice decisions, 2-by-2 grid

	Changing routes in the next round	Staying on the same route in the next round
Good outcome	c_+	s_+
Bad outcome	c_-	s_-

4. Aggregate Traffic Distribution

While the individual behavior is an interesting and valuable study in itself, the important policy-related issue is total use of each of the two routes. The estimated relationship between the individual route choice probabilities and the number of commuters on each route can be used to predict distribution of aggregate use. For this I assume that time series of aggregate use is a Markov process,

[64] Note that in some cases, the Yule coefficient is undefined. This occurs when the denominator in Equation 8 is zero.

specifically, that the probability of any potential near future (one period ahead) aggregate state (number of bridge takers at time t+1) depends only on the present aggregate state (number of bridge takers at time t). The transition probabilities (the probability of moving from one level of bridge use to another) are calculated from the estimated individual probabilities, themselves functions of the contemporaneous state. The collection of transition probabilities is a 19-by-19 (for states $N_B = 0$ through $N_B = 18$) transition matrix τ. What follows is an explanation of the derivation of τ.

For the derivation let P($N_{B_t} : N_{B_{t+1}}$) be the probability that there will be $N_{B_{t+1}}$ bridge takers tomorrow (time $t+1$) if there are N_{B_t} bridge takers today (time t). Let p_{BB}(N_{B_t}) be the probability that an individual travels on the bridge tomorrow given that he is on the bridge today and let p_{HH}(N_{B_t}) be the estimated probability that an individual on the highway today will choose to remain there tomorrow. In what follows it will be understood that the individual probabilities are functions of the state values and the notation is simplified to p_{BB} and p_{HH}. The well-known convention $q_{BB} = 1 - p_{BB}$ and $q_{HH} = 1 - p_{HH}$ is used as well. Since, in the experiment, the subjects are not permitted to communicate with each other it seems reasonable to believe that the choices are made independently. Thus, I make the assumption of independence for the calculations.

Suppose today there are seven bridge takers (N_{B_t} = 7). Then what is the probability that tomorrow there will be 5 bridge takers ($N_{B_{t+1}}$ = 5)? There are six, mutually exclusive, ways that this can happen. The probability of each is shown in Table 18. Since these represent all the possible ways there can be a transition from seven bridge takers to five, the probability of that transition is the sum of the individual ones:

(21) $$P(7:5)=\sum_{i=0}^{5}\binom{7}{5-i}p_{BB}^{5-i}q_{BB}^{2+i}\binom{11}{11-i}p_{HH}^{11-i}q_{HH}^{i}.$$

Similar calculations are made for every possible state as follows:

(22) $$P(k:m)=\sum_{i=\lambda}^{\gamma}\binom{k}{i}p_{BB}^{i}q_{BB}^{k-i}\binom{n-k}{m-i}p_{HH}^{n-k-(m-i)}q_{HH}^{m-i},$$

where $\lambda=0$ and $\gamma=m$ if $m\le k$ and $m\le 18-k$

$\lambda=m-(18-k)$ and $\gamma=m$ if $m\le k$, $m>18-k$, and $k\ge 18-k$

$\lambda=0$ and $\gamma=k$ if $k<m\le 18-k$

$\lambda=m-(18-k)$ and $\gamma=k$ if $m>k$ and $m>18-k$.

The 19-by-19 transition matrix is the ordered collection τ = [P($N_{B_t}:N_{B_{t+1}}$)].

The long-run (steady state) predicted value is a fixed point of the dynamic process encompassed in the transition matrix. The equilibrium distribution of states, d, implied by the estimated transition matrix is the one that solves d'τ = d'.

Table 18: Possible ways of 7 bridge travelers one round followed by 5 bridge travelers the next round

Number from Bridge to Highway	Number from Highway to Bridge	Probability
2	0	$\binom{7}{5} p_{BB}^{5} q_{BB}^{2} \binom{11}{11} p_{HH}^{11} q_{HH}^{0}$
3	1	$\binom{7}{4} p_{BB}^{4} q_{BB}^{3} \binom{11}{10} p_{HH}^{10} q_{HH}^{1}$
4	2	$\binom{7}{3} p_{BB}^{3} q_{BB}^{4} \binom{11}{9} p_{HH}^{9} q_{HH}^{2}$
5	3	$\binom{7}{2} p_{BB}^{2} q_{BB}^{5} \binom{11}{8} p_{HH}^{8} q_{HH}^{3}$
6	4	$\binom{7}{1} p_{BB}^{1} q_{BB}^{6} \binom{11}{7} p_{HH}^{7} q_{HH}^{4}$
7	5	$\binom{7}{0} p_{BB}^{0} q_{BB}^{7} \binom{11}{6} p_{HH}^{6} q_{HH}^{5}$

5. Individual behavior

Although there are three segments to the experiment, I look at Segment 1 for various reasons. First, this is the first segment, and so there is no transition period necessary for subjects to adjust from one segment to the next. (See Chapter I to see how transitions occur from the end of Segment 1 to the beginning of Segment 2 of this experiment.) Second, Segment 1 involves a situation with the predicted outcome of the Pigou-Knight-Downs paradox, which Arnott and Small (1994) note that this paradox "is often called 'the fundamental law of traffic congestion'" (p. 451). Thus, understanding how people behave in such an environment may help improve traffic flows in these situations.

5.1. Actual behavior

Throughout the experiment, subjects have the opportunity to switch routes between any two rounds. Thus, any subject could switch frequently, or not at all. As can be seen in Table 19, almost all subjects play on each route at least some of the time in Segment 1. Nobody travels on the highway in every round and less than 10 percent of all subjects travel on the bridge in every round. This means that almost nobody is playing the same pure strategy in every round of Segment 1.

5.2. Comparing PKD, MS, and Naïve on an individual level

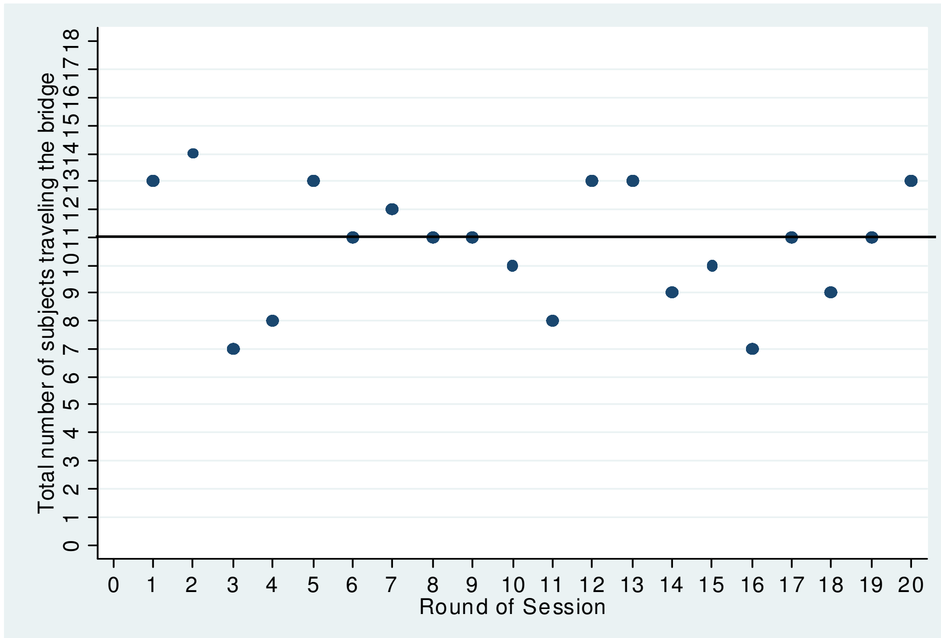
The PKD prediction states that once equilibrium is reached, nobody has an incentive to switch routes. In such a case, we would expect that no person would switch routes. However, each person may believe that some other people switch routes after a round in PKD equilibrium. This may lead to a significant portion of the population changing their route choice. This line of thinking likely occurs in this experiment, since many subjects do in fact switch routes after a round in PKD equilibrium. In fact, subjects switch routes 31.0 percent of the time in such a situation. This presents a clear rejection that once an experimental group is in PKD

Table 19: Number of times each subject travels on the bridge in Segment 1.

Number of trips on bridge	Actual distribution	Expected distribution if probability of bridge travel is $\frac{10}{17}$	Expected distribution if probability of bridge travel is $\frac{11}{18}$
0	0	0	0
1	0.012	0	0
2	0.006	0	0
3	0.006	0	0
4	0.019	0	0
5	0.037	0.002	0.001
6	0.068	0.006	0.004
7	0.043	0.018	0.011
8	0.056	0.043	0.029
9	0.062	0.082	0.061
10	0.062	0.128	0.106
11	0.099	0.167	0.152
12	0.062	0.179	0.179
13	0.074	0.157	0.173
14	0.080	0.112	0.136
15	0.062	0.064	0.085
16	0.037	0.029	0.042
17	0.062	0.010	0.016
18	0.019	0.002	0.004
19	0.062	0	0.001
20	0.074	0	0

equilibrium, that it is maintained. Figure 10 shows a round that is not in PKD equilibrium often follows a round in PKD equilibrium.

Figure 10: Round-by-round results of number of travelers on the bridge, Group 1, Segment 1



Since over 90 percent of subjects play each route at least once in Segment 1, comparing individual behavior to mixed strategies has appeal. Table 19 does this, comparing the MS and Naïve models to actual behavior by subject in all rounds of Segment 1. The actual distribution is more uniform than the MS and Naïve models predict. This suggests that subjects may have some route loyalty, but switch routes on occasion if the perceived gain is high enough. More evidence rejecting mixed-strategy play comes from Chapter I, which shows that subjects switch much less often than the MS equilibrium predicts.[65]

[65] A similar calculation shows the same result for the Naïve model.

5.3. Route Changes and Yule Coefficients

Changing routes more times within a session is correlated with higher overall point deductions in that session. This implies that subjects who frequently switch routes typically have worse outcomes than those that have a habit of travelling the same route from round to round. The correlation between the total number of road changes and the average number of points deducted is 0.24. The Spearman rank correlation of the same variables is 0.24, not independent at the 1% level. These results are similar to those found in Selten *et al* (2007).

Following Selten *et al* (2007), I assume that a Yule coefficient of at least 0.5 denotes a direct responder and –0.5 or below as a contrarian responder. Table 20 shows average Yule coefficients by group, while Table 21 shows the distribution of Yule coefficients in the segment. Almost 37% of all subjects with Yule coefficients are direct responders, while 24.2% are contrarian responders. This behavior suggests that on average, each subject is more likely to switch routes when there are more people on the route this person is traveling on. This topic is discussed in more detail in the next section, which addresses aggregate behavior.

Table 20: Average Yule coefficients by group in Segment 1.

	Segment 1
Group 1	.073 (.703) [15]
Group 2	.264 (.757) [16]
Group 3	.064 (.586) [18]
Group 4	-.032 (.737) [17]
Group 5	.045 (.730) [17]
Group 6	.192 (.725) [16]
Group 7	.123 (.728) [17]
Group 8	.267 (.682) [16]
Group 10	.191 (.650) [17]
All groups (excluding Group 9)	.133 (.689) [149]

Note: Standard deviations are in parentheses.

Number of subjects with Yule coefficients is in brackets.

Table 21: Distribution of Yule coefficients, excluding Group 9

Range	Percentage in range, Segment 1
–1.00	8.05%
–0.99 to –0.50	16.11%
–0.49 to –0.01	18.79%
0.00	3.36%
0.01 to 0.49	16.78%
0.50 to 0.99	13.42%
1.00	23.49%

6. Aggregate Behavior

No model in Section 3 seems to explain individual behavior, but this could be due in part to the variation in Yule coefficients. This leads to wonder if a mix of direct and contrarian responders leads to group behavior that appears consistent with some of the models. After showing below that the PKD outcome only predicts the average outcomes well, I show that some mixed strategies do in fact predict the aggregate outcomes quite well.

6.1. Aggregate Data Summary and the PKD Model

Table 22 reports the average number of bridge travelers per round (excluding Group 9, since only 17 subjects participate). Recall that travel time is the same on the two modes when there are 11 subjects on the bridge and 7 on the highway. All of the group averages are within 1.1 of this prediction, while none of these averages

Table 22: Average number of travelers on the bridge in Segment 1, by group, excluding Group 9

	Average *T* (Std. Dev.)
Group 1	10.70 (2.15)
Group 2	11.10 (2.05)
Group 3	*9.90* (2.45)
Group 4	10.90 (2.83)
Group 5	11.05 (1.93)
Group 6	11.30 (2.77)
Group 7	10.85 (1.98)
Group 8	10.85 (1.69)
Group 10	11.35 (2.50)
All groups (excluding Group 9)	**10.89** (2.28)

Bold denotes significantly different from predicted MS equilibrium average of 10.588 at the 10% level. Italics denote significantly different from 11 at the 10% level.

statistically differs from the PKD and Naïve average prediction (of 11 bridge travelers per round) and the MS average prediction of 10.59 at the 5 percent level.

The aggregate average of all nine groups does not significantly differ from 11, but it does differ from 10.59 at the 10 percent level. Chapter I reports the round-by-round results, with 11 subjects on the bridge observed in only about 20 percent of the rounds (35 out of 180). However, the number of people traveling the bridge often changes after a round with 11 subjects on the bridge. In fact there are only 4 out of 33 possible occurrences of one round of 11 on the bridge directly follows another. Figure 10 shows multiple instances in which a round in pure-strategy equilibrium is followed by one or more rounds out of equilibrium. This suggests that the strongest version of PKD is not operative. However, the PKD model does appear to predict the average quite well. By the Hicks criterion we should be satisfied. Although we may be satisfied with this result, other models are examined below to attempt to better explain individual behavior.

6.2. IRtCS Using Logit Estimation, and Tests of Other Models

In this subsection, I use a Logit formulation in an IRtCS framework to determine a steady state distribution. Subjects may believe that as more people travel one particular route in any round, it is more likely that the other route is the better choice in the following round. Once the Logit/IRtCS model is fit to the experimental data, we want to know what this tells us about out-of-equilibrium behavior. Specifically, we want to know how current period outcomes affect next period route choice decisions. In this case, I examine the likelihood of switching routes given the number of subjects traveling the bridge in the current round. As seen in the previous section, a majority of subjects are classified either as direct or contrarian responders, based on their Yule coefficients. These results suggest that subjects do in fact base their decisions for a particular round from the results of the previous round.

I first look at a Logit Model to estimate behavior based on the results from the previous round. After this analysis, two other models are examined in which mixed strategies in each round are not dependent on previous rounds. The MS model predicts that each commuter chooses B with probability 10/17. The other involves a strategy simpler than MS, in which each commuter plays a Naïve strategy of choosing B with probability 11/18. This mixed strategy is labeled as Naïve because it is not an equilibrium strategy but it gives an expected number of bridge travelers of 11, the same as PKD.

Table 23: Logit regressions, without a variable for the inverse of round number

	Constant	Δ
Initial choice on bridge	1.016 (11.04)	–0.090 (–3.15)
Initial choice on highway	0.361 (3.93)	0.055 (1.73)

Subject-clustered z-statistics are in parentheses.

The estimated values of the Logit Model are reported in Table 23.[66] The constants are significantly positive at the 5 percent level. The estimate of β_H is significantly positive at the 10 percent level, while the estimate for β_B is significantly negative at the 5 percent level. Both β estimates are consistent with the distribution of Yule coefficients: Subjects are more likely to switch routes after a relatively bad outcome than after a relatively good outcome. Finally, since subjects switch routes more often in early rounds,[67] Table 24 shows the same qualitative results when a control for the inverse of round number is included. Including this variable has little impact on the estimates of interest.

How do these estimates reflect on the various models of individual choice? First it is very clear from the experimental results that the simplest version of PKD cannot be true. Individuals do not settle into one route once the PKD equilibrium is achieved. Second, there is also a clear rejection of the hypothesis that travel time differential is the only determinant of choice. If this is the case the bridge parameters must be the negative of those for the highway.[68] A test imposing this

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[66] Standard errors are clustered at the individual level.

[67] Of the first nine opportunities to switch routes in Segment 1, subjects switched routes 36.0 percent of the time. Of the final 10 opportunities, subjects switched routes 30.8 percent of the time. See Roth and Erev (1995) for an examination of how experience affects behavior in experiments in three different games.

[68] If choice is modal independent then $P_{H_{t+1}|H_t} = 1 - P_{B_{t+1}|B_t}$ and $P_{B_{t+1}|B_t} = 1 - P_{H_{t+1}|H_t}$, and so the odds of remaining on the highway is the complement of the odds of remaining on the bridge. With the logit form this implies that the bridge parameters must be the negative of those for the highway.

restriction leads to rejection of modal independence, since the part of the null hypothesis related to the constant terms is rejected.

Table 24: Logit regressions, with a variable for the inverse of round number

	Constant	Δ	Inverse of round number
Initial choice on bridge	1.011	–0.090	0.024
	(9.68)	(3.17)	(0.12)
Initial choice on highway	0.565	0.056	–1.101
	(5.26)	(1.78)	(-4.48)

Subject-clustered z-statistics are in parentheses.

The CF conjecture falls to this analysis as well. If the individual probability for choosing one alternative or another is exactly 0.5 when both alternatives are equally time consuming, then the constant terms should be zero.[69] This hypothesis is clearly rejected based on the estimated z-values of the constants. The hypothesis can also be rejected by looking at the raw distribution of rounds in Segment 1 that follow rounds in equilibrium (see Table 25). In more than 90 percent of these rounds following a round in equilibrium, a majority of the drivers travel on the bridge.

Finally, I can test the hypothesis that what is observed is the result of a mixed-strategy (MS) equilibrium or Naïve strategy. If individual commuters are playing the game this way, they choose the probabilities of making either choice

[69] This is clear if one remembers that equal time cost implies that $\Delta = 0$. For the resulting choice probabilities to be 0.5 the constants of the logit probability must be zero.

Table 25: Distribution for the rounds after the previous round in equilibrium (average = 11.61)

Number of bridge commuters	Experimental Distribution
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0.061
8	0
9	0.030
10	0.182
11	0.121
12	0.333
13	0.152
14	0.030
15	0.091
16	0
17	0
18	0

prior to play, independent of traffic in the previous period. The implication for the probability model of staying on the highway in Equation 5 is that β is 0, the constant for the MS model is -0.357, and the constant for the Naïve model is -0.452. The respective predictions for the bridge model in Equation 6 are 0, 0.357, and 0.452. The constants in Table 23 are statistically different for both the MS and Naïve models at the 5 percent level. These constants being higher than the predictions of these two models implies that there is route stickiness. In other words, subjects do not switch routes as often as the MS and Naïve models predict. However, as can be seen in Table 16, the MS and Naïve models predict the aggregate results very well.

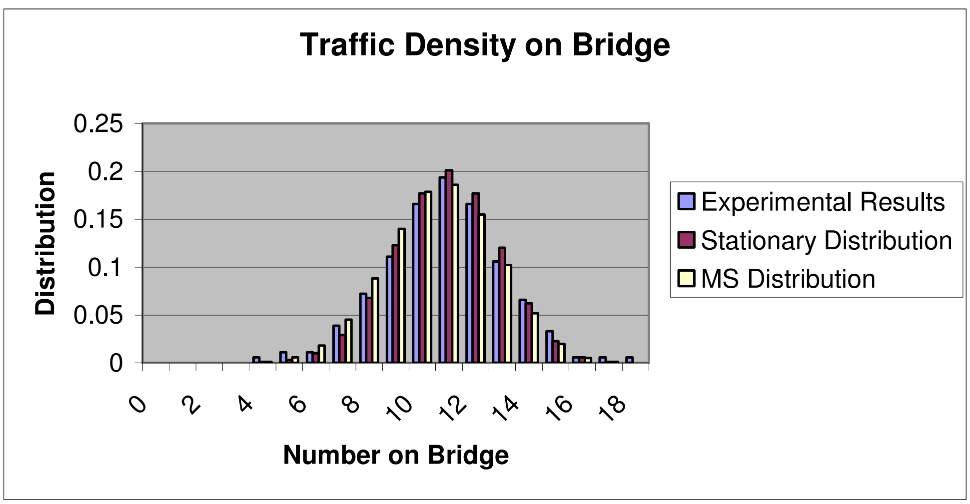
From the CF, MS, and Naïve strategies, it is assumed that the population is homogeneous with respect to the choice decision rules. This hypothesis is not a good assumption to make on the individual level, as the Yule coefficients show heterogeneity in the subject pools of this paper and Selten *et al* (2007). However, as seen below, the homogeneity assumption works well in the predictive power of a subject group for some of the mixed-strategy models.

6.3. Predictive power of each model

The MS and Naïve models also make predictions about the observed traffic distribution. If, as specified by MS, the probability of bridge choice is 10/17, then the probability of each traffic level is easily calculated as binomial probabilities. A similar calculation holds true for Naïve with probability of bridge travel of 11/18. The outcomes of these calculations are exhibited in Table 16.

The aggregate comparisons are remarkable. First, although the Naïve model is rejected on the individual level, the average bridge traffic is very close to the Naïve prediction. Furthermore, both the Logit Steady State distribution and the MS distribution have approximately the same average values. In addition, both follow the experimental outcome quite closely (See Figure 11). It appears from Table 16 that both the Logit Steady State and the MS and Naïve distributions are approximately identical to the experimental outcome. It seems that even though the particular models are rejected when tested against data on individual behavior, they each do well in predicting aggregate outcomes.

Figure 11: Experimental Traffic Density, Stationary Distribution, and MS Distribution



A more formal way of measuring how well a model fits the aggregate outcomes is the sum of square differences (SSD). If $F(i)_{exp}$ denotes the experimental frequency of i subjects traveling the bridge, and $F(i)_{\alpha}$ denotes the experimental frequency of the predicted distribution of α (which could be either PKD, Logit Steady State, MS, CF, or Naïve), then the SSD is defined as

(23) $$SSD = \sum_{i=0}^{18} (F(i)_{\alpha} - F(i)_{\exp})^2 .$$

Of the five models, all except the PKD and CF do well in predicting the aggregate outcomes. As seen in Table 16, the Naïve model does the best, with an SSD of 0.0008. The Logit Steady State model has the next lowest SSD, followed by the MS. Finally, the CF model does much better than the PKD prediction, since the CF model at least factors in that subjects do not necessarily stay on the same route once the PKD equilibrium is reached.

Table 26: Sum of square differences of distribution in Table 25

Model	Sum of square differences
PKD	0.9532
Stationary distribution	0.0516
MS	0.0642
Coin flip	0.1532
Naïve Decision	0.0496

Table 26 shows the SSD of the various models in rounds immediately following the PKD equilibrium of 11 subjects traveling the bridge. The results are similar comparatively to the overall case, with the Naïve, Logit Steady State, and mixed-strategy models doing relatively well at predicting the outcomes, followed by the CF model. The PKD model does very poorly at predicting rounds after the PKD equilibrium. Thus, although the PKD equilibrium predicts the average number of travelers on each route quite well, it would be unrealistic to assume that the PKD equilibrium actually occurs from one commute to the next in a real commuting situation.

7. Conclusion

If, as Hicks asserts, the job of the economist is not to predict what Mr. Brown and Mr. Jones will do, but for our "…hypothesis only … [to have] *prima facie* plausibility when it is applied to a statistical average" (p. 55). In this case each of the suggested models that produces a distribution of possible outcomes passes that test. Specifically, although each individual does not behave with a perfectly mixed strategy, nor is necessarily aware that changing routes is due in part to the number of travelers on each route in each round, assuming so can adequately explain aggregate behavior. Such modeling is useful when examining actual commuting environments, since policy makers usually care about aggregate results and not those of any individual driver.

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Appendix A[70]

Let p_i equal the probability of person i traveling the bridge and let $p\_$ equal the probability of everybody except person i traveling the bridge. If each subject is risk neutral, then maximizing expected utility is equivalent to maximizing expected payout. These are also equivalent to minimizing total expected travel time in Segment 1. Let n_B denote the number of travelers on the bridge in any round.

In Segment 1, if person i travels the highway, the travel time is guaranteed at 20 minutes, while the expected travel time on the bridge is

(A1) $E(n_B \mid \text{person } i \text{ travels the bridge}) = 1 + 17p\_$.

To find out when person i is indifferent on either route, $p\_$ must be found such that expected travel times are equal:

(A2) $9 + (1 + 17p\_) = 20$,

which yields

(A3) $p\_ = \dfrac{10}{17} = 0.588.$

Thus, when the expected travel times are equal, person i is indifferent over any strategy. In such a case, choosing $p = 10/17$ gives a symmetric mixed-strategy Nash equilibrium.

In the mixed-strategy equilibrium for Segment 1 described above, the expected number of travelers on each route is 18 * (10/17) = 10.588. This results in fewer expected travelers on the bridge than in the pure-strategy equilibrium. The

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[70] This Appendix uses the same techniques as Selten *et al* (2007).

variance of this distribution is

(A4) $V_p = p\,(1 - p) = 0.242,$

which results in a standard deviation of 2.088 total travelers in each period.

If T_H and T_B denote the total number of expected travelers over 20 rounds on the highway and bridge, respectively, then

(A5) $T_B = 20 \times 18p = 211.765$ and $T_H = 20 \times 18(1 - p) = 148.235.$

The variance of the totals in equation (A5) is

(A6) $V = 87.197,$

which results in the variance of the mean of 9 groups[71] of

(A7) $$\frac{V}{9} = 9.689\,.$$

The standard error is then

(A8) $\sigma = 3.113.$

In the experiment, a per-group average of 217.778 bridge trips are taken in the 9 groups with 18 subjects, while the expected number of trips in the mixed-strategy equilibrium is 211.765. Since this difference is about 1.93σ from the mean, a null hypothesis of subjects playing the mixed strategy equilibrium cannot be rejected at the 5% level.

Similarly, in Segment 2, $p_i = p\_ = 4/17$, $V_p = 0.180$, $T_B = 84.706$, $T_H = 275.294$, $V = 64.775$, $(V / 9) = 7.197$, and $\sigma = 2.683$. In the experiment, a per-group average of 113 bridge trips are taken in the 9 groups with 18 subjects, while the

[71] Group 9 is not examined here, since it only has 17 subjects.

expected number of trips in the mixed-strategy equilibrium is 84.706. This difference is more than 10σ from the mean, and so a null hypothesis of subjects playing the mixed strategy equilibrium can be rejected at a very high level of significance.[72]

Another aspect worth examining is whether or not the predictions of mixed-strategy equilibrium are consistent with the number of route changes in the experiment. For each subject playing the mixed-strategy equilibrium in Segment 1, the probability q that a subject will switch routes from one round to the next is

(A9) $q = 2p(1 - p) = 0.484.$

In Segment 1, there are 19 opportunities to switch routes, leading to 342 opportunities to switch routes for all players within the same group of Segment 1. The expected number of route changes (R) is thus

(A10) $R = 342q = 165.7.$

Since the binomial distribution is used, the variance is

(A11) $V_q = q(1 - q) = 0.2498,$

which implies that the variance of R is

(A12) $V_R = 342V_q = 85.42.$

Since there are 9 groups of participants with 18 subjects, it is useful to calculate the variance for the mean of 9 observations:

[72] Since there is likely a transition period from Segment 1 to Segment 2, it is worth noting that the same rejection of the null hypothesis can be made when only the final 10 rounds of Session 2 are looked at.

(A13) $\frac{V_R}{9} = 9.491.$

The standard error of this variance is thus

(A14) $\sigma_R = 3.081.$

The observed number of route changes per group is 113.78. This is more than $16\sigma_R$ below the predicted number of route changes, leading to the conclusion that the number of route changes being inconsistent with the mixed-strategy Nash equilibrium.

In Segment 2, the calculations are similar, except the right hand side of Equation A2 is 14 instead of 20 (since the toll is equivalent to six minutes of travel time). This leads to a mixed-strategy equilibrium of all subjects on the bridge with probability 4/17, or 0.235. The calculations for the other variables result in q = 0.360, R = 123.1, V_q = 0.2304, V_R = 78.78, $(V_R / 9)$ = 8.754, and σ_R = 2.959. The number of route changes per experimental group is 80.56, which is more than $14\sigma_R$ below the predicted number of route changes. Again, the number of route changes is not consistent with the mixed-strategy Nash equilbrium.

Appendix B

In this section, I show the set of possible pure strategy Nash equilibria. Recall that in each case described below, there are three people of each type. The two experimental designs are point deductions of 8, 9, 10, 11, and 12, or 4, 7, 10, 13, and 16.

In the framework of per-minute point deductions ranging from 8 to 12, suppose that 5 or fewer subjects on the bridge constitutes an equilibrium. Then there must be at least one person with an 11 or 12 point per minute deduction on the highway. If an 11 is on the highway, her point deduction for traveling this route is 25×11, or 275. She could switch routes to the bridge and have a point deduction no higher than $70 + 17 \times 11$, or 257. Since the bridge is strictly the better choice when 5 or fewer other subjects are on the bridge, each 11 is better off by traveling the bridge. Similarly for 12's, the choices are 25×12, or 300, and $70 + 17 \times 12$, or 274, respectively. Again, each 12 is better off on the bridge. This leads to a contradiction, since all 11's and 12's cannot be on the bridge when there are 5 or fewer on this route. So no equilibrium is possible such that 5 or fewer on the bridge.

Next, suppose that an equilibrium exists such that 8 or more subjects travel the bridge. This means that at least one 8, 9, or 10 must be traveling on the bridge. Then if an 8 is on the bridge, his point deduction on this route is at least $70 + 19 \times 8$, or 222. He could switch to the highway for a point deduction of 25×8, or 200. Thus, each 8 is better off on the highway in this situation. Similarly for 9's, the choices are $70 + 19 \times 9$, or 241, and 25×9, or 225, respectively. For the 10's, the choices are $70+19\times10$, or 260, and 25×10, or 250, respectively. Both the 9's and 10's are better off on the highway. This leads to a contradiction, since all 8's, 9's, and 10's cannot be on the highway if there are 8 or more subjects on the bridge.

Next suppose that an equilibrium exists such that exactly 6 subjects are on the bridge:

- Suppose there is an 8 on the bridge. Then her point deduction on this route is $70+17\times8$, or 206. She could switch to the highway for a point deduction of 25×8, or 200. She is better off on the highway, and thus any 8 must be on the highway whenever there is an equilibrium with exactly 6 subjects on the bridge.
- Suppose there is a 9 on the bridge. Then his point deduction on this route is $70+17\times9$, or 223. He could switch to the highway for a point deduction of 25×9, or 225. So there could be a 9 on the bridge since switching to the highway is not a better option.
- Suppose there is a 9 on the highway. Then his point deduction on this route is 25×9, or 225. He could switch to the bridge for a point deduction of $70+18\times9$, or 232. So there could be a 9 on the highway, since switching to the bridge is not a better option.
- Suppose there is a 10 on the bridge. Then her point deduction on this route is $70+17\times10$, or 240. She could switch to the highway for a point deduction of 25×10, or 250. So there could be a 10 on the bridge since switching to the highway is not a better option.

- Suppose there is a 10 on the highway. Then her point deduction on this route is 25×10, or 250. She could switch to the bridge for a point deduction of $70+18\times10$, or 250. So there could be a 10 on the highway since switching to the bridge is not a better option.
- Suppose there is an 11 on the highway. Then his point deduction on this route is 25×11, or 275. He could switch to the bridge for a point deduction of $70+18\times11$, or 268. So he is better off on the bridge, and thus any 11 must be on the bridge whenever there is an equilibrium with exactly 6 subjects on the bridge.
- Suppose there is a 12 on the highway. Then her point deduction on this route is 25×12, or 300. She could switch to the bridge for a point deduction of $70+18\times12$, or 286. So she is better off on the bridge, and thus any 12 must be on the bridge whenever there is an equilibrium with exactly 6 subjects on the bridge.
- In summary: All 11's and 12's must be on the bridge, the 9's and 10's could be on either route, and all 8's must be on the highway for any equilibrium with exactly 6 subjects on the bridge. From these criteria, the only equilibrium possible under these conditions is if all 11's and 12's are on the bridge, and all others are on the highway.

Next suppose that an equilibrium exists such that exactly 7 subjects are on the bridge:

- Suppose there is an 8 on the bridge. Then his point deduction on this route is $70+18\times8$, or 214. He could switch to the highway for a point deduction of 25×8, or 200. He is better off on the highway, and thus any 8 must be on the highway whenever there is an equilibrium with exactly 7 subjects on the bridge.
- Suppose there is a 9 on the bridge. Then her point deduction on this route is $70+18\times9$, or 232. She could switch to the highway for a point deduction of 25×9, or 225. She is better off on the highway, and thus any 9 must be on the highway whenever there is an equilibrium with exactly 7 subjects on the bridge.
- Suppose there is a 10 on the bridge. Then his point deduction on this route is $70+18\times10$, or 250. He could switch to the highway for a point deduction of 25×10, or 250. So there could be a 10 on the bridge since switching to the highway is not a better option.
- Suppose there is a 10 on the highway. Then his point deduction on this route is 25×10, or 250. He could switch to the bridge for a point deduction of $70+19\times10$, or 260. So there could be a 10 on the highway since switching to the bridge is not a better option.
- Suppose there is an 11 on the bridge. Then her point deduction on this route is $70+18\times11$, or 268. She could switch to the highway for a point deduction of 25×11, or 275. So there could be an 11 on the bridge since switching to the highway is not a better option.

- Suppose there is an 11 on the highway. Then her point deduction on this route is 25×11, or 275. She could switch to the bridge for a point deduction of $70+19\times11$, or 279. So there could be an 11 on the highway since switching to the bridge is not a better option.
- Suppose there is a 12 on the highway. Then his point deduction on this route is 25×12, or 300. He could switch to the bridge for a point deduction of $70+19\times12$, or 298. He is better off on the bridge, and thus any 12 must be on the bridge whenever there is an equilibrium with exactly 7 on the bridge.
- In summary: All 12's must be on the bridge, the 10's and 11's could be on either route, and all 8's and 9's must be on the highway for any equilibrium with exactly 6 subjects on the bridge. From these criteria, the only equilibrium possible under these conditions is if a total of 4 10's and 11's are on the bridge and the other 2 are on the highway.

In the framework of per-minute point deductions ranging from 4 to 16, most of the results are similar. There is no equilibrium with 5 or fewer on the bridge, nor are there any equilibria with 8 or more on the bridge. In any equilibrium with exactly 6 on the bridge, all 13's and 16's must be on the bridge. This means that all 4's, 7's, and 10's must be on the highway. In any equilibrium with 7 on the bridge, all 13's and 16's must be on the bridge and all 4's and 7's must be on the highway. Then any equilibrium with 7 on the bridge must have exactly 1 10 on the bridge and the other 2 on the highway.