

## DUOPOLY PRICES UNDER CONGESTED ACCESS

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forthcoming in *Journal of Regional Science*

### Abstract

Consider two firms, at different locations, supplying a homogenous good at constant marginal production cost. Consumers incur travel costs to the firm for each unit purchased, and the travel costs increase with the amount of travel to each firm (congestion). When all traffic and all congestion are generated by travel to a duopolist, both the Nash-Bertrand equilibrium prices and the Nash-Cournot equilibrium prices exceed the sum of the marginal production cost and the marginal external travel cost. However, when the road is shared by travelers to the duopolists' facilities and travelers in competitive markets, the Nash-Bertrand duopoly price equals the competitive price and the Nash-Cournot price contains a markup.

JEL: D43, D62, L13, R10, R41

Keywords: Duopoly, oligopoly, imperfect competition, transport, congestion

### Acknowledgments

This research is supported financially by the U.S. Department of Transportation and the California Department of Transportation through the University of California Transportation Center. I thank Ralph Braid, Jan Brueckner, Bruno De Borger, Ami Glazer, Jun Ishii, Ken Small, the editor and three anonymous referees for valuable discussions and/or remarks on earlier versions. I am indebted to a referee for bringing Braid's (1986) working paper, which closely resembles parts of this paper, to my attention. Ralph Braid chose not to co-author this paper, but it has strongly benefited from his work. Errors and shortcomings are my responsibility.

## 1. INTRODUCTION

This paper analyzes how duopolistic providers of perfect substitutes set prices when access to each duopolist's facility is subject to congestion and the costs of access are borne by consumers departing from a common location. The pricing rules are derived for both Nash-Bertrand and Nash-Cournot behavior. The model has a natural spatial interpretation: consumers may travel to suppliers at several locations to obtain an identical good; they incur the time and money costs of travel, and travel is subject to road congestion. I distinguish between the case where all congestion is related to access to the duopolists' facilities and the case of mixed traffic. Under mixed traffic, the road is simultaneously used by travelers in the duopoly markets and by travelers in a different market, which I assume to be competitive.

Consider, for example, competition between airports in large metropolitan areas like the San Francisco Bay Area. Passengers often can depart for the same destination at the same time, with the same airline and for the same fare<sup>1</sup>, from different airports (San Francisco, San Jose, or Oakland). Access costs to the airport then co-determine the choice for a particular airport (Pels et al., 2000), especially on short haul flights (e.g. to Southern California) where access costs form a large share of the total trip cost. For some origins in the Bay Area, one of these alternatives always dominates the others, but for others two of the three airports are a reasonable option, so that the problem can be analyzed as a duopoly.<sup>2</sup> Clearly, access to the airport often takes place under congested conditions, certainly in peak-periods, and this affects the time costs of access. Airport authorities may directly charge passengers for airport use, or the charges can be reflected in airfares.

How will profit-maximizing airport managers charge for access to the airport under the conditions just described?<sup>3</sup> This paper analyzes how access prices depend on congestion experienced in accessing each airport, abstracting from other features such as differences in distance. As will be seen, the access charge depends on the type and the extent of competition with other airports, and on whether congestion experienced when accessing the airport is airport-specific (duopoly traffic only) or is subject to transport network congestion (mixed traffic). Using a stylized model of pricing when access to spatially dispersed suppliers of an identical good is subject to congestion, I derive the pricing rules in case the market is a Nash-Bertrand duopoly (price competition) and a Nash-Cournot duopoly (quantity competition). The locations of firms and consumers are given, and they are connected by congestible links in a simple network. I find that duopoly-specific congestion generates market power both with price and quantity competition, and that the presence of general network congestion allows duopolists to charge markups with quantity competition, but not with price competition. Other examples of congestion-prone facilities that may compete in an oligopolistic setting include national parks (Richardson, 2002), ports, swimming pools, museums, etc.

The structure of the paper is as follows. Section two reviews relevant literature, section three develops the model, section four discusses a numerical illustration, and section five concludes.

## 2. LITERATURE

The interaction between congestion and oligopoly pricing has been the subject of earlier studies. Braid (1986) derives the Nash-Bertrand and the Nash-Cournot pricing rules for a congestion-prone symmetric duopoly. Scotchmer (1985a,b) analyzes price

competition for congestible facilities, assuming fixed total demand, and considers the optimal number of facilities (whereas I keep the number of competitors fixed). De Palma and Leruth (1989) consider a two-stage game in which two firms supplying a perfect substitute first choose capacity and then compete in prices, and find that congestion relaxes price competition. The effect of the presence of non-duopoly traffic is neglected in all mentioned studies.

In a broader sense, this paper relates to research that analyzes congestion externalities when distortions exist in the economy. One part of that literature focuses on the impact of constraints on pricing instruments in the transportation sector; another studies the impact of tax distortions outside the transport sector on optimal transport pricing. Imperfect competition is a further source of distortions, but has received relatively little attention in the analysis of congestion pricing. I list some exceptions.

First, several studies look at the implications of private ownership of road infrastructure under various types of market structure on tolling and investment decisions (e.g. de Palma and Lindsey, 2000; Edelson, 1971); they confirm the general insight that private owners will at least partly internalize the congestion externality if the market structure allows. Second, a number of industrial organization analyses focus on the relation between congestion and market power in the airline industry (e.g. Brueckner, 2002; Pels and Verhoef, 2004) and the electrical power industry (e.g. Borenstein, 2000). However, since in these networks firms or network operators decide on the network flows (but not final consumers), the nature of the interaction between congestion and market power is different from that in road transport networks (where consumers themselves distribute flows over the network), as considered here. Third, some papers on spatial

economics focus on the relation between imperfect competition and location choice, usually abstracting from congestion externalities (cf. Gabszewicz and Thisse, 1996, and Fujita and Thisse, 2002, for overviews). My paper keeps location fixed and assesses the relation between congestion and market power. It could be seen as a short to medium term analysis, as compared to the more long run scope of the endogenous location models. Lastly, my analysis focuses on ‘mill pricing’, that is: markets where consumers incur the access costs. This assumption is motivated by the examples given above (airports, national parks, etc.), where consumers need to travel to the supplier to consume the good. The case of delivered pricing entails different strategic interactions and different outcomes (cf. Dastidar, 1995, 1997, and Vives, 1999).

### 3. THEORY

The theoretical analysis in the following subsections considers a duopoly (with extensions to oligopoly), but the model also is interpretable as a duopsony (or oligopsony), which would fit the situation where consumers can travel to two or more employment locations. So, the analysis captures the interaction between congested travel and access prices as well as the interaction between congested commuting and wages. The analysis focuses entirely on internal solutions. The first subsection derives Nash-Bertrand and Nash-Cournot pricing rules for the case where all traffic is duopoly-related; the second subsection adds interaction with traffic related to other (competitive) markets. The third subsection considers generalizations to oligopoly, discusses limiting cases, and deals with interactions between monopoly and duopoly markets.

## Duopoly Traffic Only

Consider a transport network consisting of two links that connect the single trip origin to two firms,  $A$  and  $B$ , as in Figure 1. The duopolists supply a perfect substitute, so that consumer demand is the sum of both firms' output. Denote demand for perfect substitutes that are supplied by two suppliers at fixed locations by  $q^D$ . There is strict complementarity between trips and purchases: buying one unit of the good or service requires one trip (the consequences of relaxing this assumption are discussed in the section on mixed traffic).<sup>4</sup> Denoting the duopoly market by superscript  $D$  and location by subscripts  $A$  and  $B$ , I have  $q^D = q_A^D + q_B^D$ . Each firm's marginal production cost  $c_i^D$ ,  $i = A, B$  is constant. The duopolists' prices are  $p_A^D$  and  $p_B^D$ . Travel to the firm is costly and paid for by the consumer. As there is – for now – only duopoly traffic, and given the strict complementarity between demand and trips, the traffic volume on a link is the sum of all trips over the link. Travel costs per person,  $a_i$ , on each link increase with link volume  $q_i$ ,  $i=A,B$ :  $a_i = a_i[q_i]$ ,  $a'_i \equiv \frac{\partial a_i}{\partial q_i} > 0$ ,  $i = A, B$ .<sup>5</sup> Under the standard assumption that travelers neglect the increase in travel times that they cause to other road users, the congestion externality is given by  $a'_i q_i$ ,  $i = A, B$ .

The generalized price,  $g^D$ , is the sum of time costs and prices. In case consumers buy goods at both locations (interior solution), the generalized price is<sup>6</sup>:

$$(1) \quad g^D = a_A[q_A^D] + p_A^D = a_B[q_B^D] + p_B^D$$

Consumer demand declines with the generalized price:  $q^D = q^D[g^D]$ ,  $\frac{\partial q^D}{\partial g^D} < 0$ . The

inverse demand function is denoted by  $G[q^D]$ , and its derivative  $\left(\frac{\partial G}{\partial q^D} \equiv G'\right) < 0$ .

In the duopoly market, prices will depend on specific assumptions concerning market structure. I subsequently discuss the cases of Nash-Bertrand (price) competition and Nash-Cournot (quantity) competition. The pricing rules for these cases are obtained by combining the appropriate set of first-order conditions for the general problem formulated in (2). The first term of the Lagrangian is the objective (profits). The two constraints require that marginal willingness to pay equals the generalized price in the duopolistic market at both locations. Since we assume an interior solution, this requires equality of generalized prices across locations, as in (1).

$$(2) \quad \mathfrak{L}_A = (p_A^D - c_A^D)q_A^D + \lambda^1 \left\{ G[q_A^D + q_B^D] - p_A^D - a_A[q_A^D] \right\} + \lambda^2 \left\{ G[q_A^D + q_B^D] - p_B^D - a_B[q_B^D] \right\}$$

Price competition. Assume first that the duopolists compete in prices, meaning that one firm takes the other firm's price as fixed but recognizes that quantities adjust to maintain consumer equilibrium. Hence, taking partial derivatives with respect to  $p_A^D, q_A^D, q_B^D$  leads to the first order conditions:

$$(3) \quad q_A^D = \lambda^1$$

$$(4) \quad p_A^D - c_A^D + \lambda^1 (G' - a'_A) + \lambda^2 G' = 0$$

$$(5) \quad \lambda^1 G' + \lambda^2 (G' - a'_B) = 0$$

Combining (3) and (4) leads to

$$(6) \quad p_A^D = c_A^D - q_A^D (G' - a'_A) - \lambda^2 G'$$

Substituting (3) into (5), solving for  $\lambda_2$  and substituting the result into (6) gives:

$$(7) \quad p_A^D = c_A^D + a'_A q_A + q_A G' \frac{a'_B}{G' - a'_B}$$

Equation (7) implicitly defines the reaction function for the duopolist at  $A$ . Since the marginal social cost consists of the constant marginal resource cost and the marginal external congestion cost (the first two terms on the right-hand side of (7)), and the third term on the right-hand side of (7) is positive, it follows that the Nash-Bertrand equilibrium price exceeds the marginal social cost. The intuition for this result is as follows. First, in contrast to the standard Nash-Bertrand model, undercutting a competitor does not allow capturing the entire market because access costs increase. The demand curve facing the firm is continuous and is downward sloping rather than perfectly elastic, and this allows a markup. Second, each duopolist internalizes the marginal external congestion cost of travel to its facility. Third, each firm charges a markup above the marginal social cost that increases in the congestibility of access to the competing facility and decreases in the price-elasticity of demand.

In other words, congestion generates market power because the decision of a consumer to switch from, e.g., facility  $A$  to facility  $B$  raises the generalized price at  $B$  and reduces that to  $A$ . So, when firm  $A$  increases its price, the generalized price at  $A$  increases but so does the generalized price at  $B$ . As congestion is an externality, consumers neglect the interaction between prices and generalized prices, and this generates market power. This contrasts with the purely competitive market, where the externality is not reflected in prices. Note that, while Pigouvian tolls are required to internalize the externality in the purely competitive market, in the purely duopolistic case the prices are too high, so that a subsidy would be required to attain Pareto-efficiency.<sup>7</sup>



Price competition limits market power in the Nash-Bertrand case with duopoly traffic only. When the elasticity of demand decreases ( $|G'| \rightarrow \infty$ ), the Nash-Bertrand mark-up rises; in a symmetric model, it does not exceed the total external congestion cost in the network. When demand becomes very elastic ( $G' \rightarrow 0$ ), the Nash-Bertrand mark-up converges to the marginal external congestion cost on the link to each firm. Perfectly elastic demand hence forces the firms to internalize the externality, that is, to charge the marginal social cost of producing and transporting the good. In the absence of congestion, the standard Nash-Bertrand outcome of competitive pricing is obtained, clarifying the conclusion that congestion generates market power under Nash-Bertrand competition.

Quantity competition. Consider next duopolists who compete in quantities. Within my set-up, this means replacing the first-order condition (5) by the partial derivative of the Lagrangian in (2) with respect to  $p_B^D$ , as the duopolist at A now takes  $q_B^D$  as fixed. The other conditions do not change. Hence, instead of (5) I get (8).

$$(8) \quad \lambda^2 = 0$$

Using (8) in (6) leads to

$$(9) \quad p_A^D = c_A^D + a'_A q_A^D - q_A^D G'$$

As in the Nash-Bertrand case, the Nash-Cournot markup implied by (9) exceeds the marginal external congestion cost on the link to the own location. In contrast to the Nash-Bertrand case, the dependence of the price at one location on congestion to the other is indirect, through the market shares and the local elasticity of demand.

Comparing the Nash-Bertrand and the Nash-Cournot prices at equal demand levels, it can

be seen that the Nash-Cournot markup is always larger than the Nash-Bertrand markup, because the Nash-Cournot markup does not directly depend on the slope of the travel cost function to the other location.

When  $|G'| \rightarrow \infty$  (decreasing elasticity of demand), also  $p_A^D \rightarrow \infty$ ; in contrast to the case of price competition, the network interaction does not limit markups under quantity competition. With an increasing demand elasticity ( $G' \rightarrow 0$ ), the price converges to the marginal social cost ( $p_A^D \rightarrow c_A^D + a'_{AA} q_A^D$ ) under quantity competition, as under price competition.

### Mixed Traffic

In the previous section, all traffic is generated by the duopolists, so that all congestion is specific to the duopoly market. In this section, consumers at a single location still can travel to facilities in two locations,  $A$  and  $B$ , but now at each location two goods are supplied,  $C$  and  $D$ ; cf. Figure 2. Good  $C$  is supplied under competitive conditions, so that the price is equal to the (constant) marginal resource cost. The motivation for choosing a competitive market structure for the non-duopolistic market is that when more than one supplier of a particular good is present at each location, the competitive outcome results in the model of the previous subsection. The market for good  $D$  is duopolistic, with a single supplier at each location.

Access to each location is subject to congestion: travel times ( $a_i, i=A,B$ ) increase with traffic volumes. There is strict complementarity between travel volumes and demand volumes in both markets, so that traffic volumes on each link are the sum of demand in both markets served by that link. In contrast to the previous subsection, I

assume that the constant marginal resource costs are independent of the location in each market.

The resource cost in the duopolistic market is  $c^D$ ; the marginal resource cost in the competitive market equals the market price  $\bar{p}^C$ . Let the (downward sloping) demand functions in both markets be  $q^C = q^C[g^C]$  and  $q^D = q^D[g^D]$  where  $g^C$  and  $g^D$  indicate the generalized price, defined as the sum of link time costs  $a_i, i = A, B$  and store prices  $p_i^j, i = A, B$  and  $j = C, D$ . This formulation assumes that demand for the goods is independent, except for the interaction through congestion.<sup>8</sup> The inverse demand functions are  $H \equiv H[q^C]$  and  $G \equiv G[q^D]$ , where  $q^C = q_A^C + q_B^C$  and  $q^D = q_A^D + q_B^D$ . The derivatives are denoted  $H'$  and  $G'$ . I shall consider an interior solution, with sales of both goods at both locations. The duopolist at location  $A$  then solves the following problem.

$$\begin{aligned}
 \mathfrak{S}_A = & (p_A^D - c^D)q_A^D \\
 & + \lambda^1 \left\{ G[q_A^D + q_B^D] - p_A^D - a_A[q_A^C + q_A^D] \right\} \\
 (10) \quad & + \lambda^2 \left\{ G[q_A^D + q_B^D] - p_B^D - a_B[q_B^C + q_B^D] \right\} \\
 & + \lambda^3 \left\{ H[q_A^C + q_B^C] - \bar{p}_A^C - a_A[q_A^C + q_A^D] \right\} \\
 & + \lambda^4 \left\{ H[q_A^C + q_B^C] - \bar{p}_B^C - a_B[q_B^C + q_B^D] \right\}
 \end{aligned}$$

The first term and the first two constraints are the same as in (2), except that travel times now depend on travel associated with both markets. The last two constraints require that there is consumer equilibrium in the competitive market.

Price competition. Assume first that the duopolists compete in prices. Taking partial derivatives of (10) with respect to  $p_A^D, q_A^D, q_B^D, q_A^C, q_B^C$  leads to the first-order conditions:

$$(11) \quad q_A^D = \lambda^1$$

$$(12) \quad p_A^D - c^D + \lambda^1(G' - a'_A) + \lambda^2 G' - \lambda^3 a'_A = 0$$

$$(13) \quad \lambda^1 G' + \lambda^2(G' - a'_B) - \lambda^4 a'_B = 0$$

$$(14) \quad -\lambda^1 a'_A + \lambda^3(H' - a'_A) + \lambda^4 H' = 0$$

$$(15) \quad -\lambda^2 a'_B + \lambda^3 H' + \lambda^4(H' - a'_B) = 0$$

Combining (11) and (12) leads to

$$(16) \quad p_A^D = c^D - q_A^D(G' - a'_A) - \lambda^2 G' + \lambda^3 a'_A.$$

Adding (13) and (14), and using (11), produces

$$(17) \quad q_A^D(G' - a'_A) + \lambda^2(G' - a'_B) + \lambda^3(H' - a'_A) + \lambda^4(H' - a'_B) = 0.$$

Observe that the left-hand side of (15), which equals zero, appears in (17). The remainder of (17) then also equals zero, cf. (18).

$$(18) \quad q_A^D(G' - a'_A) + \lambda^2 G' - \lambda^3 a'_A = 0.$$

Finally, since (18) appears on the right-hand side of (16), I conclude that:

$$(19) \quad p_A^D = c^D.$$

Equation (19) says that the duopolist at location A charges the marginal resource cost. Of course, a similar result holds for location B. Therefore, in the interior solution, the presence of a competitive market forces the price-competing duopolists to charge

competitive prices. Intuitively, the arbitrage taking place through the competitive market effectively makes the access cost exogenous to the duopolist, so that the demand curve facing each duopolist becomes perfectly elastic. Since congestion can no longer be manipulated strategically, the model reverts to the standard Nash-Bertrand result of competitive pricing.

Note that, as in the standard Nash-Bertrand model, the distribution of demand between both duopolists (and between both competitive locations) is indeterminate, so that a sharing rule is required to determine the final outcome. In other words, whereas the consumer equilibrium constraint acted as a sharing rule in the model with only duopoly traffic, this is not so with mixed traffic.

Quantity competition. Consider next duopolists competing in quantities. As in the previous subsection this requires replacing the first-order condition with respect to  $q_B^D$  by the first-order condition with respect to  $p_B^D$ , as the duopolist at A now takes  $q_B^D$  as fixed. The other conditions do not change. Hence, instead of (13) I have:

$$(20) \quad \lambda^2 = 0$$

Using (11) and (20) in (12) leads to

$$(21) \quad p_A^D = c^D - q_A^D (G' - a'_A) + \lambda^3 a'_A$$

Using (11) and (20) in (14) and in (15) produces (22) and (23):

$$(22) \quad -a'_A q_A^D + \lambda^3 (H' - a'_A) + \lambda^4 H' = 0$$

$$(23) \quad \lambda^3 H' + \lambda^4 (H' - a'_B) = 0$$

Solving (23) for  $\lambda^4$ , substituting in (22), solving for  $\lambda^3$ , and substituting in (21) gives the pricing rule:

$$(24) \quad p_A^D = c^D + (a'_A - G')q_A^D + a'_A q_A^D \left( \frac{a'_A}{H' - a'_A - H' \frac{H'}{H' - a'_B}} \right)$$

The price under quantity competition clearly differs from the competitive price despite the presence of competitive traffic. In particular, the first two terms on the right-hand side of (24) are the same as in the case of duopoly traffic only, cf. (9), and the last term is negative. Consequently, the Nash-Cournot markup is smaller under mixed traffic than in the case where there is only duopoly traffic. This difference in markups is smaller as access to the facility at  $A$  is more congestion-prone and as  $H'$  is smaller, that is, competitive demand is more price-elastic.

As long as the solution is an interior one, the presence of a competitive market with identical resource costs at both locations forces access costs to be equal across locations. In contrast to the case where all traffic is duopoly traffic, this forces the duopolists' prices to be equal, also when congestion conditions differ between both facilities. As differences in access conditions do not matter (only overall network congestion does), the duopolists will not take them into account when deciding on the quantities, and therefore they will equally split the market. The quantities in the competitive market will differ across locations, allowing overall equilibrium to be reached.

When the elasticity of demand goes to infinity in the duopoly market ( $G' \rightarrow 0$ ) and to zero in the competitive market ( $|H'| \rightarrow \infty$ ), then the Nash-Cournot duopoly price converges to the marginal social cost. With  $G' \rightarrow 0$  and  $H' \rightarrow 0$ , the bracketed expression on the right-hand side of (24) converges to -1. Consequently, the duopoly

price approaches the competitive price. This says that, when demand is very elastic in both markets, market power is completely eroded due to the interaction of competitive and duopoly-related traffic on the network. As in the case of pure duopoly traffic, the price approaches infinity as the elasticity of demand in the duopoly market decreases.

Relaxing strict complementarity. Up to now, purchasing a unit of the good required a trip to either duopolist (strict complementarity). This assumption simplifies the analysis, but it is extreme. Some insight on how results change if the complementarity assumption is relaxed, can be obtained from a reinterpretation of the mixed market analysis. Assume that all traffic is duopoly traffic, but that consumers can choose between two modes to travel to a firm. One mode is congestible ('cars') but the other is not ('subway'), so that the generalized access cost using subway is fixed (i.e. not dependent on traffic volumes). In that case, if the consumer is only interested in accessing a duopolist as cheaply as possible, then the equilibrium access cost is equal to the generalized price of the subway (which I assume to be equal across destinations), as long as both modes are used. This is a strong relaxation of the complementarity between access and the creation of congestion: if an uncongested mode is available, access will not increase congestion, even if all access requires a trip. The consequence is that the pricing rules of the mixed traffic case are reproduced: with price competition there will be no mark-up, but with quantity competition there will.

However, it is not realistic to assume that an uncongested mode is available, especially during peak hours in metropolitan areas. Public transport modes get congested, because they share use of the road network, or because demand approaches

capacity. Therefore, in as far as access to the duopolists contributes to increasing congestion of the urban transport system, the connection between congestion and prices remains intact, but is likely to be quantitatively less strong than in the ‘cars only’ setting.

### Oligopoly

When, instead of two firms, there are  $N$  oligopolists and each of them is connected to the consumers’ location by a separate congestible link, the price rules are found by simple extensions of the approach used above. When all traffic is oligopoly traffic, the Lagrangian for the case with  $N$  firms is as follows:

$$\mathfrak{S}_A = (p_A^D - c_A^D)q_A^D + \sum_{i=1}^N \lambda_i^D (G - p_i^D - a_i)$$

For the case of price competition, the first-order conditions with respect to  $p_A^D, q_A^D, q_j^D, j \neq A$  are:

$$q_A^D = \lambda_A^D$$

$$p_A^D = \lambda_A^D a'_A - G' \sum_{i=1}^N \lambda_i^D$$

$$G' \sum_{i=1}^N \lambda_i^D - \lambda_j^D a'_j = 0, \forall j \neq A$$

I restrict attention to the symmetric model where  $c_i^D = c^D$  and  $a'_i = a', i = 1, \dots, N$ .

Solving for the Nash-Bertrand pricing rule for firm  $A$  then produces:

$$p_A^D = c^D + a' q_A^D \left( \frac{a' - NG'}{a' - (N-1)G'} \right).$$

As  $N$  becomes very large, the bracketed expression approaches one, and the duopoly price approaches the marginal social cost. In this symmetric equilibrium, prices and quantities are equal at all locations ( $p_i^D = p^D$  and  $q_i^D = q^D, i = 1, \dots, N$ ).



When there is only duopoly traffic and the firms are Nash-Cournot competitors, the conditions with respect to  $q_j^D, j \neq A$  are replaced by those with respect to  $p_j^D, j \neq A$ :

$$\lambda_j^D = 0, \forall j \neq A$$

The resulting price rule takes the same form as the one for the duopoly case:

$$p_A^D = c_A^D - q_A^D (G' - a'_A)$$

When there is mixed traffic, the oligopolist at location  $A$  determines the critical points for profit maximization from:

$$\mathfrak{S}_A = (p_A^D - c_A^D)q_A^D + \sum_{i=1}^N (\lambda_i^D (G - p_i^D - a_i) + \lambda_i^C (H - p_i^C - a_i))$$

When the oligopolists compete in prices, it is intuitively clear that the price rule is the same as before: all firms charge marginal costs.

$$p_A^D = c^D$$

For the case of quantity competition for mixed traffic, the price rule is implied by the first-order conditions with respect to  $p_A^D, q_A^D, p_j^D, \forall j \neq A$  and  $q_k^C, k = 1, \dots, N$ :

$$q_A^D = \lambda_A^D$$

$$p_A^D = c^D - G' \sum_{i=1}^N \lambda_i^D + (\lambda_A^D + \lambda_A^C) a'_A$$

$$\lambda_j^D = 0, \forall j \neq A$$

$$H' \sum_{i=1}^N \lambda_i^C - (\lambda_k^D + \lambda_k^C) a'_k = 0, k = 1, \dots, N$$

Restricting attention to a fully symmetric model, the first three of these conditions lead to the same expression as for the duopoly case:

$$p_A^D = c^D + (a' - G')q_A^D + \lambda_A^C a'$$

A solution for  $\lambda_A^C$  can be found from the fourth set of conditions. In the symmetric model, it is clear that  $\lambda_j = \lambda_A, \forall j \neq A$ . Some algebra then allows writing the price rule as follows:

$$p_A^D = c^D - q_A^D (G' - a') - a' \frac{a' q_A^D}{a' - H' - \frac{H' H' (N-1)}{a' - (N-1)H'}}$$

As in (24) the presence of traffic in competitive markets reduces oligopolists' ability to charge markups. The larger  $N$ , the more market power is eroded, and the markup converges to the marginal external congestion cost as  $G' \rightarrow 0$ . Note that also in this symmetric equilibrium prices and quantities are equal at all locations ( $p_i^D = p^D$  and  $q_i^D = q^D, i = 1, \dots, N$ ).

Note that for a given demand curve, increasing the number of spatially separated oligopolists means that trips become spread thin over the available links, so that congestion converges to zero as well. The pure Nash-Bertrand model then converges to a competitive outcome, and the markup in the Nash-Cournot model depends on the elasticity of demand alone, irrespective of  $N$ .

Lastly, I have assumed up to now that all consumers can choose between the two (or  $N$ ) facilities. How are the results affected when some consumers can choose to buy or not buy at one facility, but the other one is never chosen? Obviously, when the duopolists can price-discriminate between consumers that do and do not have a choice between facilities, the price rules derived above continue to hold for consumers that do have a choice, and a monopoly price is charged to consumers that do not have a choice between facilities. The monopoly price charged facility at A to consumers that will never

choose B is:  $p_A^M = q_M(a'_A - G')$ . When the duopolists cannot price discriminate between consumer types but do know the demand functions for both types, they will charge a weighted average of the duopoly price and the monopoly price, as long as all traffic is duopoly-related. When there is mixed traffic and the duopolists are Nash-Bertrand competitors, the competitive price will be charged in all markets (including the one where there is a monopoly).

#### 4. NUMERICAL ILLUSTRATION

This section clarifies the intuition and the relative importance of the various parameters that determine the results derived in section 3. To this end I use a numerical example that, while not referring to any specific real-world situation, uses reasonable orders of magnitude for the demand and access cost functions. After briefly discussing the construction of the example, I present results for a central (symmetric) scenario, a scenario where access cost functions differ between locations, and a scenario that varies the relative importance of the duopolistic and the competitive market.

The example is a linear version of the models depicted in Figures 1 and 2. Table 1 summarizes the parameterizations for the central scenario, which is fully symmetric. The demand functions in the pure duopoly case and mixed markets case are constructed such that, for intercepts that are equal in both markets, the slopes lead to equal traffic volumes for the competitive benchmark solution. The intercept and the slope of the access cost functions is chosen so that the marginal external congestion cost is nearly as large as the marginal private access cost. Access costs are a large share of the generalized price in all equilibria (between 25% and 100%). In the competitive solution

3,119 trips are made to each location. The competitive prices equal the marginal resource costs, which are set at zero, and the access cost is 39.<sup>9</sup> In the case of mixed traffic the trips are equally distributed over the competitive and the duopolistic market.

Table 2 shows that, in case all traffic is duopoly-related, total demand is highest in the competitive solution (column III), followed by the efficient solution (where prices equal the marginal social cost in the equilibrium; IV), the Nash-Bertrand equilibrium (I) and the Nash-Cournot equilibrium (II). Both the Nash-Bertrand and the Nash-Cournot price exceed the marginal external cost of congestion (which in the example equals the marginal social cost). The markup allows positive profits.

Next, the efficient traffic volumes are the same when there is only duopoly traffic (IV) and when there is mixed traffic (VIII), by construction. When there is mixed traffic, the Nash-Bertrand equilibrium (V) is identical to the competitive outcome (VII). The Nash-Cournot price still exceeds the marginal social cost, so that demand in the duopoly market (VI) is less than socially optimal. Demand in the competitive market, however, is larger than the socially optimal level. In this example, total traffic happens to be slightly larger than the socially optimal level.

Table 3 illustrates the consequences of introducing asymmetrical access cost functions in the model. The intercepts (interpretable as free flow travel costs) and the slope of the function leading to B are unchanged, but the slope of the function for the link leading to location A is varied. This amounts to changing the travel cost conditions of the entire network, while demand functions do not change. When all traffic is duopoly-related, the main effect of the asymmetry is to increase the profits of the facility that is less congestion-prone, and decrease the profits of the relatively more congestion-prone

facility. This holds for both the Nash-Bertrand and the Nash-Cournot model. But the decrease of profits for the more congestion-prone facility is larger under Nash-Bertrand competition, whereas the increase of profits for the less congestion-prone facility is larger under Nash-Cournot competition.<sup>10</sup>

Note that when all traffic is generated by the duopoly, the prices at both locations differ with asymmetric congestion functions. The more congestion-prone facility is in a weaker competitive position, so is forced to reduce prices; the opposite holds for the less congestible facility. Of course, the generalized prices still are equal at both locations as the equilibrium is an interior one.

When traffic is mixed, the effects of the asymmetry differ. First, in interior solutions the Nash-Bertrand model reduces to the competitive model, also under asymmetry. The equilibrium conditions only determine the distribution of aggregate traffic flows over the network, but not the demand in each of the four markets. I assume equal sharing of the market between the Nash-Bertrand competitors, but this is a random assumption for the mixed traffic Nash-Bertrand model. Next, under Nash-Cournot competition, the presence of a competitive market with equal marginal resource costs requires the Nash-Cournot prices to be equal at both locations. It then is the best mutual response for the Nash-Cournot competitors to supply equal quantities. Consequently, total demand in the competitive market is asymmetrically distributed over both locations when the access cost functions differ. In particular, when accessing location  $A$  becomes relatively cheap, a larger share of total competitive demand is served at location  $A$ . As a consequence of this interaction between the competitive market and the Nash-Cournot duopoly, the Nash-Cournot profits are equal at both locations (given equal marginal

resource costs), irrespective of how congestion-prone access to the facility is. Also, if the Nash-Cournot quantity decision is interpreted as a capacity decision, then differences in accessibility that are caused by congestion do not affect the capacity choice if there is competitive traffic.

Lastly, Table 4 shows what happens to the Nash-Bertrand and the Nash-Cournot outcomes when the inverse demand curve in the competitive market is shifted downwards and the inverse demand curve in the duopolistic market is simultaneously shifted upwards, keeping the traffic flow and the marginal external congestion cost the same as in the central scenario. The competitive and the efficient outcomes are not reported, as they are not affected by this experiment. As is clear from the theoretical result, the type of asymmetry considered has no impact on the Nash-Bertrand equilibrium. For the Nash-Cournot case, increasing the intercept of the duopoly demand function has the effect of increasing the equilibrium duopoly price. In other words, the Nash-Cournot markup increases as duopoly traffic is a larger share of total traffic, keeping the equilibrium traffic flow constant.

## 5. CONCLUSION

Congested access to oligopolistic suppliers of perfect substitutes is a source of markups. When all traffic is generated by demand in the oligopolistic market, the equilibrium prices exceed marginal social costs, both under Nash-Bertrand and under Nash-Cournot competition. When the road network is shared by trips for competitive markets and for the oligopolistic market, the Nash-Bertrand markups disappear and the Nash-Cournot markups are reduced.

The analysis has focused on the effect of congested access on duopoly prices. Of course, in realistic settings many other factors will determine access charges, for example at airports. Some relevant extensions include the representation of consumer heterogeneity and product differentiation, the introduction of distance as a determinant of travel costs, an explicit consideration of multi-modal access, adding capacity choices, and explicit analysis of the conditions for interior solutions (which could interact with pricing and capacity decisions). Nevertheless, the interactions identified in this paper will matter in those more general contexts, and as traffic network congestion is substantial, taking them into account is not of mere theoretical interest.

TABLE 1: Demand and Cost Functions for the Central Scenario

	Duopoly traffic only		Mixed traffic	
	Competitive market	Duopolistic market	Competitive market	Duopolistic market
Intercept inverse demand function	-	195	195	195
Slope inverse demand function	-	-0.025	-0.05	-0.05
Intercept access cost function	1.617		1.617	
Slope access cost function	0.012		0.012	
Marginal resource costs	0	0	0	0



TABLE 2: Central Scenario: Symmetric Equilibria

		I	II	III	IV	V	VI	VII	VII
		Duopoly traffic only				Mixed traffic			
		Bertrand	Cournot	Competition	Efficient	Bertrand	Cournot	Competition	Efficient
Demand at A, B	Duopoly	2,355.2	1,953.4	3,119.1	2,613.3	1,559.5	1039.7	1,559.5	1306.6
	Competitive	-	-	-	-	1,559.5	1615.2	1,559.5	1306.6
Traffic flow to A, B		2,355.2	1,953.4	3,119.1	2,613.3	3,119.1	2654.9	3,119.1	2613.3
Generalized Price A, B	Duopoly	77.2	97.3	39.0	64.3	69.0	91.0	69.0	64.3
	Competitive	-	-	-	-	39.0	33.5	39.0	64.3
Price A, B	Duopoly	47.4	72.3	0	31.4	0	57.6	0	31.4
	Competitive	-	-	-	-	0	0	0	31.4
Time cost A, B	Duopoly	29.9	25.1	39.0	33.0	39.0	33.5	39.0	33.0
MECC <sup>a</sup> A, B		28.3	23.4	37.4	31.4	37.4	31.9	37.4	31.4
Profits A, B	Duopoly	111,541.0	141,178.0	0	81,591.0 <sup>b</sup>	0	59,839.0	0	40,975.5 <sup>b</sup>
	Competitive	-	-	-	-	0	0	0	0

<sup>a</sup> Marginal external congestion cost

<sup>b</sup> I randomly assign the proceeds from the markup to profits, as profits and consumer surplus have equal weights when the social objective is to be efficient.

TABLE 3: Asymmetrical Access Cost Functions

		Asym.: slope A=0.006, slope B=0.012				Central (Sym.: slope A=slope B=0.012)				Asym.: slope A=0.024, slope B=0.012			
		Pure		Mixed		Pure		Mixed		Pure		Mixed	
		Bert.	Cournot	Bert.	Cournot	Bert.	Cournot	Bert.	Cournot	Bert.	Cournot	Bert.	Cournot
Demand A	Duop.	3,092.2	2,391.1	2,778.5	1,111.4	2,355.2	1,953.4	1,559.5	1,039.7	1,614.1	1,429.9	488.3	976.7
	Comp.	-	-	1,667.1	2,648.1	-	-	1,559.5	1,615.2	-	-	1,465.0	696.0
Demand B	Duop.	2,156.0	1,805.5	555.7	1,111.4	2,355.2	1,953.4	1,559.5	1,039.7	2,498.7	2,130.2	2,441.7	976.7
	Comp.	-	-	1,667.1	768.4	-	-	1,559.5	1,615.2	-	-	1,465.0	2,368.7
Gen. Price	Duop.	63.8	90.1	28.3	83.9	77.2	97.3	39.0	91.0	92.2	106.0	48.5	97.3
	Comp.	-	-	28.3	24.2	-	-	39.0	33.5	-	-	48.5	41.8
Price A	Duop.	43.6	74.1	0	59.7	47.4	72.3	0	57.6	51.8	70.1	0	55.6
	Comp.	-	-	0	0	-	-	0	0	-	-	0	0
Price B	Duop.	36.3	66.8	0	59.7	47.4	72.3	0	57.6	60.6	78.8	0	55.6
	Comp.	-	-	0	0	-	-	0	0	-	-	0	0
Time cst A		20.2	16.0	28.3	24.2	29.9	25.1	39.0	33.5	40.4	35.9	48.5	41.8
Time cst B		27.5	23.3	28.3	24.2	29.9	25.1	39.0	33.5	31.6	27.2	48.5	41.8
Profits A	Duop.	134,895	177,232	0	66,335	111,541	141,178	0	59,839	83,651	100,182	0	54,274
Profits B	Duop.	78,275	120,613	0	66,335	111,541	141,178	0	59,839	151,369	167,900	0	54,274

TABLE 4: Demand and Cost Functions for the Central Scenario

		Central		Asymmetric intercepts	
		Bertrand	Cournot	Bertrand	Cournot
Intercept demand func.	Duopoly	195	195	292.5	346.5
	Competitive	195	195	97.5	97.5
Demand at A, B	Duopoly	1,559.5	1,039.7	2534.5	2014.8
	Competitive	1,559.5	1,615.2	584.5	640.1
Traffic flow to A, B		3119	2654.9	3119	2654.9
Generalized Price A, B	Duopoly	39.0	91.0	39.0	145.0
	Competitive	39.0	33.5	39.0	33.5
Price A, B	Duopoly	0	57.6	0	111.5
	Competitive	0	0	0	0
Time cost A, B		39.0	33.5	39.0	33.5
MECC* A, B		37.4	31.8	37.4	31.8
Profits A, B	Duopoly	0	59,839	0	224,750
	Competitive	0	0	0	0

FIGURE 1: Model Structure with Only Duopoly Traffic

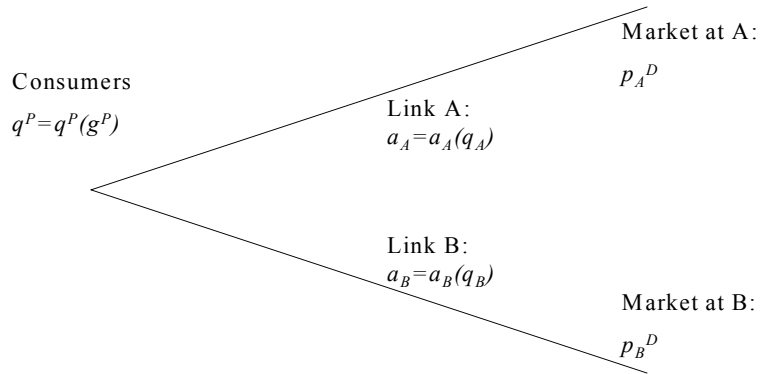
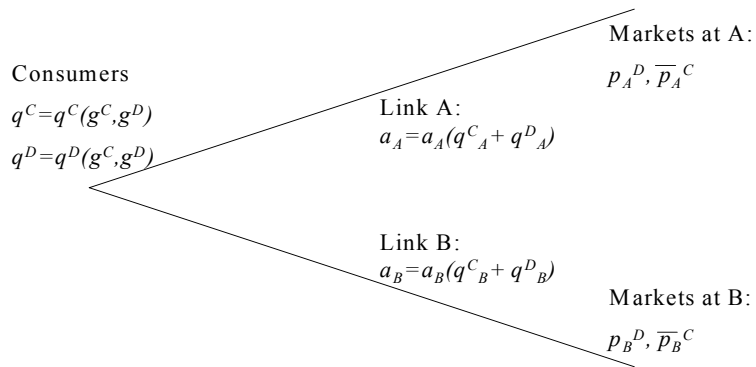


FIGURE 2: Model Structure with Mixed Traffic



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<sup>1</sup> Casual inspection of published fares suggests that this is a reasonable assumption. However, published fares and actual fares may differ because of quantity restrictions pertaining to the published fares.

<sup>2</sup> In a survey of passengers departing from the Bay Area in 1995 (MTC, 1996), some 70% of respondents at Oakland Airport and San Jose Airport state that they have a choice between airports; the share for San Francisco Airport (45%) is lower because of the higher share of intercontinental flights. Usually two of the three airports are substitutes (SFO and OAK, or SFO and SJO), but not all three.

<sup>3</sup> Direct passenger charges are less ubiquitous in the US than elsewhere (Pels et al., 2000). However, in a deregulated environment direct passenger charges may be expected to play a more prominent role.

<sup>4</sup> As each consumer makes one trip to either firm for each unit purchased, the demand curve can be viewed as the aggregation over consumers with a different willingness to pay for the good, but with equal and constant marginal values of time. If each consumer buys one or zero units, this is consistent with the standard Hotelling approach to recovering a continuous aggregate demand function from discrete individual demands. Note that, with a single consumer, there is no congestion externality. More in general, I assume  $N$  consumers, and normalize  $N$  to 1.

<sup>5</sup> Distance could be included as an argument of the travel cost function. Doing so is important in an applied analysis of the interactions described in the theory, as such an applied analysis will need to take explicit account of corner solutions, and the occurrence of such corner solutions is affected by differences in distance. The present paper focuses on the interaction between congestion and market power; simplifying the analysis by using assumptions guaranteeing that such an interaction exists and is the only issue of concern.

<sup>6</sup> In case goods are purchased at one location only, the price at that location cannot be larger than the price at the other.

<sup>7</sup> Van Dender (2002) discusses the optimal toll expressions in some detail.

<sup>8</sup> The results presented below for the Nash-Bertrand case continue to hold when the goods are substitutes or complements, but the Nash-Cournot results are affected.

<sup>9</sup> All costs and prices can be rescaled without affecting the results presented here, so I do not specify units.

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<sup>10</sup> This is hard to infer from Table 3, but is confirmed in simulations with more extreme asymmetries. The results of these are available on request.