

# Robust Maintenance Policies for Markovian Systems under Model Uncertainty

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### **Abstract**

Asset management systems help public works agencies decide when and how to maintain and rehabilitate infrastructure facilities in a cost effective manner. Many sources of error, some difficult to quantify, can limit the ability of asset management systems to accurately predict how built systems will deteriorate. This paper introduces the use of robust optimization to deal with epistemic uncertainty. The Hurwicz criterion is employed to ensure management policies are never 'too conservative.' An efficient solution algorithm is developed to solve robust counterparts of the asset management problem. A case study demonstrates how the consideration of uncertainty alters optimal management policies and shows how the proposed approach may reduce maintenance and rehabilitation (M&R) expenditures.

# 1 Introduction

The United States has historically made an extraordinary investment in its infrastructure. For instance, according to the GAO (2001) the federal government has spent an average of about \$59 billion annually since the 1980s on the nation's civilian infrastructure. The emphasis of infrastructure investment has shifted in the past 30 years toward maintenance rather than new construction. According to the CBO (1999), a larger and larger proportion is being spent on maintenance of the total expenditure on public works improvements, with the proportion of public non-capital spending for infrastructure increasing from 39% in 1960 to 57% in 1994. However, the magnitude of maintenance and rehabilitation (M&R) investment has been far from sufficient. Therefore, the critical issue facing public works agencies today is how to allocate limited resources that are available for M&R so as to obtain the best return for their expenditure.

Asset management is the process by which agencies monitor and maintain built systems of facilities, with the objective of providing the best possible service to the users, within the constraints of available resources. More specifically, the asset management process refers to the set of decisions made by a public works agency concerning the allocation of funds among a system of facilities and over time. The primary decisions made by a public works agency are the selection and scheduling of M&R actions to perform on the facilities in the system during a specified planning horizon.

Asset management systems are tools to help public works agencies with these M&R decisions. Experience with asset management systems in the United States shows that the benefits of these systems have been substantial in practice. For example, the OECD (1987) has reported that the Arizona Department of Transportation has saved over \$200

million in M&R costs over a five-year period following the implementation of their Pavement Management System (PMS). These savings are achieved because the M&R decisions are made by the PMS with an objective to minimize the life-cycle costs of the pavement sections in the network.

The cost minimization problem solved by the PMS is an asset management problem. In that case, a network of facilities is being managed. Asset management problems may also be formulated at the level of individual facilities. One example of such an optimization problem is presented in Formulation 1. Formulation 1 may be solved via dynamic programming. Value or policy iteration techniques may be employed to find both an optimal management policy ( $a^*$ ) and optimal future management costs ( $v$ ).

Formulation 1: Single Facility Long Term Asset Management Markov Decision Problem

**Model Parameters:**

Let  $\alpha$  be the discount rate factor.

Let  $I$  be the set of condition states for the asset.

Let  $A$  be the set of management actions that may be performed on the asset.

Let  $i^*$  in  $I$  be the initial state of the asset to be managed.

Let  $c$  in  $I \times A \rightarrow \Re$  relate condition state, action pairs to the sums of corresponding agency and user costs.

Let  $p$  in  $I \times I \times A \rightarrow [0,1]$  relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the action is taken.

**Decision Variables:**

Let  $v$  in  $I \rightarrow \Re_{\geq 0}$  relate condition states to optimal (least) future expected discounted costs.

Let  $a^*$  in  $I \rightarrow A$  relate condition states to optimal actions to take.

**Mathematical Program:**

*Minimize* $_{v,a^*} v(i^*)$

such that

$$1. v(i) = c(i, a^*(i)) + \alpha[\sum_{j \in I} p(i, j, a^*(i))v(j)] \quad \forall i \in I$$

Note that even for the relatively simple Formulation 1, a large amount of error-free data is required as input in order to develop efficient M&R policies. The most important of these data items relate to the condition of the facility. There are two forms of information on infrastructure condition: information on current condition, provided by facility inspection, and information on future condition, provided by the forecast of a deterioration model. Deterioration models are mathematical relations having as a dependent variable the condition of the facility and as independent variables the facility's age, current condition, level of utilization, environment, historical M&R actions etc. Both forms of condition information are characterized by a large degree of uncertainty. Inspection output has a number of errors from a variety of sources: technological limitations, data processing errors, errors due to the nature of the infrastructure surface inspected, and errors due to environmental effects. These sources of errors interact and produce measurement biases and random errors. If the magnitudes of the biases are known, then the measurements can be corrected for their presence by suitable subtraction and multiplication. Humplick (1992) has shown, in contrast, that the random errors can only be described in terms of the parameters of their statistical distributions, if known, and can not be corrected for. On the other hand, model forecasts are associated with a high degree of uncertainty due to the following factors:

- Exogenous factors such as the environment and level of utilization;
- Endogenous factors such as facility design and materials;
- Statistical factors such as the limited size and scope of data sets used to generate models.

Although we can improve the quality of data by developing more advanced inspection

methods and deterioration models, it is impossible to eliminate entirely the uncertainty associated with M&R decision-making. In state-of-the-art asset management systems, the stochastic nature of a facility’s deterioration process (intrinsic uncertainty) has been captured through the use of stochastic process models as representations of facility deterioration. On the other hand, the determination of the parameters of these stochastic models is still subject to significant uncertainty. This is what is known as epistemic uncertainty, uncertainty due to lack of knowledge.

Statistical testing by Prozzi and Madanat (2004) revealed statistically significant unobserved heterogeneity in their model that “cannot be ignored” despite the fact that the model’s prediction error was 50% smaller than that of earlier models developed with the same data. Unobserved heterogeneity cannot be corrected because it is due to factors that cannot readily be measured or accounted for such as the quality of construction, the material composition of facilities, and the performance of the mixed materials. Thus, both logical reasoning and empirical evidence indicate the presence of epistemic uncertainty in infrastructure deterioration modeling.

## **2 Robust Optimization**

Robust optimization is employed to address the epistemic uncertainty associated with M&R decision-making. Robust optimization is a modeling methodology to solve optimization problems in which the data are not known precisely but known to be in certain ranges, or to belong to certain sets (“uncertainty sets”). The approach is to seek optimal (or near optimal) solutions that are not overly sensitive to any realization of uncertainty. Recent

reviews on this topic can be found in Mulvey et al (1995), Ben-Tal and Nemirovski (2002) and El Ghaoui (2003) among others.

In robust dynamic programming, no underlying stochastic model of the data is assumed to be known. A robust feasible solution is one that tolerates changes in the problem data, up to a given bound known a priori, and a robust optimal solution is a robust feasible solution with the best possible value of the objective function. By carefully constructing and efficiently solving the robust counterpart of the original problem, it is possible to obtain solutions that gracefully trade off performance vs. guaranteed robustness and reliability.

Robust optimization will lead to a new generation of decision-support tools that facilitate the solution of decision problems with uncertainty. Successful applications can be found in many areas, such as finance, telecommunication, structural engineering, transportation etc. No research has been performed to apply robust optimization to asset management. More importantly, previous studies on robust optimization have mainly focused on solving uncertain linear, conic quadratic and semidefinite programming problems as in El Ghaoui and Lebret (1997), El Ghaoui et al. (1998), and Ben-Tal and Nemirovski (1999). Relatively little research has been done on the subject of robust dynamic programming. El Ghaoui and Nilim (2002) and Iyengar (2002) are two related studies on this topic.

A sample robust optimization problem is presented in Formulation 2. It represents one way that the optimization problem presented in Formulation 1 might be reformulated to consider epistemic uncertainty.

In Formulation 2, an “uncertainty level” between 0 and 1 is employed. Setting the uncertainty level to 0 implies no uncertainty, meaning an uncertainty set is defined that includes only the transition probability matrix given by an initial model. An uncertainty set is a set of

transition probability matrices, each of which may define the system in question. Increasing the uncertainty level adds new transition probability matrices to the set. In this example, a transition matrix will be included in the uncertainty set if and only if the difference between any element of the transition matrix and the corresponding element of the original matrix is less than or equal to the uncertainty level. Seen in this light, the uncertainty level represents how large an error in transition probabilities is considered possible. An uncertainty level of 0 would correspond to assuming a known transition probability matrix accurately and precisely defines an infrastructure asset's decay. On the other hand, an uncertainty level of 1 corresponds to a complete lack of confidence in any given transition probability matrix.

Formulation 2: A MAXIMIN Robust Version of Formulation 1

**New Model Parameters:**

Let  $d$  be the uncertainty level.

Let  $q$  in  $I \times I \times A \rightarrow [0,1]$  relate initial condition state, final condition state, action triples to the initially assumed model of probabilities of immediately transitioning between the two states after the management action is taken.

**New Decision Variables:**

Let  $p$  in  $I \times I \times A \rightarrow [0, 1]$  relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, as considered in the robust optimization.

**Mathematical Program:**

*Minimize* $_{v,a^*}$  *Maximize* $_p v(i^*)$

such that

1.  $v(i) = c(i, a^*(i)) + \alpha[\sum_{j \in I} p(i, j, a^*(i))v(j)]$   $\forall i \in I$
2.  $|p(i, j, a) - q(i, j, a)| \leq d$   $\forall i, j \in I$  and  $a \in A$
3.  $\sum_{j \in I} p(i, j, a) = 1$   $\forall i \in I$  and  $a \in A$
4.  $p(i, j, a) \geq 0$   $\forall i, j \in I$  and  $a \in A$

Note that the approach being used in Formulation 2 is a MAXIMIN approach. Benefits

(costs) are maximized (minimized) considering that nature will act as an opponent. The majority of work in the field of robust optimization uses such an approach. Planning agencies may perceive such an approach to be too conservative. When managing a large network of facilities, it may be too costly, and unrealistic, to manage each one under the assumption that nature is always malevolent. An alternate approach, known as MAXIMAX, involves acting under the assumption that nature will work with decision makers instead of against them. The most realistic point of view would be to recognize that nature will act neither as an adversary nor as an ally, but somewhere in between.

One attractive alternative involves applying the Hurwicz criterion, as found in Revelle et al (1997). The Hurwicz criterion allows a decision maker to set his or her own 'optimism level.' The optimism level,  $\beta$ , must be a number between 0 and 1. The pessimism level is defined as  $1 - \beta$ . Decisions are then made by selecting actions that maximize benefits obtained by summing the optimism level times the greatest possible benefit level with the pessimism level times the least possible benefit level. An optimism level of 0 would correspond to minimizing costs assuming deterioration is the most severe of all the rates of deterioration considered possible. This decision criterion would be equivalent to MAXIMIN. An optimism level of 1 would correspond to minimizing costs assuming deterioration is the least severe considered possible. All optimism levels between 0 and 1 trade off costs in the most severe case with costs in the least severe case. In the context of asset management, a robust optimization problem that employs the Hurwicz criterion might be defined as in Formulation 3.

Formulation 3: A Hurwicz Criterion Robust Version of Formulation 1

**New Model Parameters:**

Let  $\beta$  be the optimism level.

**New Decision Variables:**

Let  $p_1$  in  $I \times I \times A \rightarrow [0,1]$  relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, as considered in MAXIMAX robust optimization.

Let  $p_2$  in  $I \times I \times A \rightarrow [0,1]$  relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, as considered in MAXIMIN robust optimization.

Let  $v_1$  in  $I \rightarrow \mathbb{R}$  relate condition states to future expected discounted costs, as defined given the transition probabilities considered in MAXIMAX robust optimization.

Let  $v_2$  in  $I \rightarrow \mathbb{R}$  relate condition states to future expected discounted costs, as defined given the transition probabilities considered in MAXIMIN robust optimization.

**Mathematical Program:**

$Minimize_{v_1, v_2, a^*} [\beta Minimize_{p_1} [v_1(i^*)] + (1 - \beta) Maximize_{p_2} [v_2(i^*)]$   
such that

1.  $v_1(i) = c(i, a^*(i)) + \alpha [\sum_{j \in I} p_1(i, j, a^*(i)) v_1(j)]$   $\forall i \in I$
2.  $v_2(i) = c(i, a^*(i)) + \alpha [\sum_{j \in I} p_2(i, j, a^*(i)) v_2(j)]$   $\forall i \in I$
3.  $|p_1(i, j, a) - q(i, j, a)| \leq d$   $\forall i, j \in I$  and  $a \in A$
4.  $|p_2(i, j, a) - q(i, j, a)| \leq d$   $\forall i, j \in I$  and  $a \in A$
5.  $\sum_{j \in I} p_1(i, j, a) = 1$   $\forall i \in I$  and  $a \in A$
6.  $\sum_{j \in I} p_2(i, j, a) = 1$   $\forall i \in I$  and  $a \in A$
7.  $p_1(i, j, a) \geq 0$   $\forall i, j \in I$  and  $a \in A$
8.  $p_2(i, j, a) \geq 0$   $\forall i, j \in I$  and  $a \in A$

Using the Hurwicz criterion lets decision makers adjust the optimism level and create management policies as optimistic as they choose. It still might be possible to characterize Formulation 3 as too conservative on the grounds that certain transitions might be consid-

ered in the MAXIMIN part of Formulation 3, even though such transitions are considered impossible in real life. It is a relatively simple task to ensure that certain 'impossible' transitions are never considered in robust optimization. Formulation 4 includes two constraints that ensure that any transitions with zero probability in the original model are assigned zero probability in any model used in the robust optimization.

Formulation 4: A Constrained Hurwicz Criterion Robust Version of Formulation 1

**Mathematical Program:**

$Minimize_{v_1, v_2, a^*} [\beta Minimize_{p_1} [v_1(i^*)] + (1 - \beta) Maximize_{p_2} [v_2(i^*)]$   
such that

1.  $v_1(i) = c(i, a^*(i)) + \alpha [\sum_{j \in I} p_1(i, j, a^*(i)) v_1(j)]$   $\forall i \in I$
2.  $v_2(i) = c(i, a^*(i)) + \alpha [\sum_{j \in I} p_2(i, j, a^*(i)) v_2(j)]$   $\forall i \in I$
3.  $|p_1(i, j, a) - q(i, j, a)| \leq d$   $\forall i, j \in I$  and  $a \in A$
4.  $|p_2(i, j, a) - q(i, j, a)| \leq d$   $\forall i, j \in I$  and  $a \in A$
5.  $p_1(i, j, a) [1 - sign(q(i, j, a))] = 0$   $\forall i, j \in I$  and  $a \in A$
6.  $p_2(i, j, a) [1 - sign(q(i, j, a))] = 0$   $\forall i, j \in I$  and  $a \in A$
7.  $\sum_{j \in I} p_1(i, j, a) = 1$   $\forall i \in I$  and  $a \in A$
8.  $\sum_{j \in I} p_2(i, j, a) = 1$   $\forall i \in I$  and  $a \in A$
9.  $p_1(i, j, a) \geq 0$   $\forall i, j \in I$  and  $a \in A$
10.  $p_2(i, j, a) \geq 0$   $\forall i, j \in I$  and  $a \in A$

where  $sign(x) = \frac{x}{|x|}$  if  $x$  is non-zero and 0 if  $x = 0$ .

Constraints 5 (and 6) guarantee that the elements in  $p_1$  (and  $p_2$ ), corresponding to a zero element in  $q$ , will be set to zero.

The overall optimization is not as complex as it might appear at first glance. The final

formulation, the constrained Hurwicz criterion asset management problem, just combines the costs associated with best case and worst case transition probability matrices. The simplest way to solve this problem is to first solve problems of finding best and worst case transition probabilities and associated cost-to-go functions for all potential policies in all states. The problems of finding best and worst case transition probability matrices are themselves separable into smaller problems of finding transitions leaving initial state-action pairs. Transition probability matrices are included in the uncertainty set if and only if their every term is within the uncertainty level of some given matrix. Thus the constraint of being in the uncertainty set is “separable” into individual constraints on individual probabilities. Note that only constraints 8 and 9 link different terms from the transition probability matrix. Transition probabilities are linked by the fact that they must together form a complete transition probability matrix, so the sum of the probabilities from any initial state-action pair must be 1. In any given state  $i$ , acting under policy  $a$ , maximizing costs just implies finding a solution to the objective function  $Maximize_p[v(i)] = Maximize_p[c(i, a(i)) + a \sum_{j \in I} p(i, j, a(i))v(j)]$  which can be reduced to  $Maximize_p \sum_{j \in I} [p(i, j, a(i))v(j)]$ . This maximization problem can be solved exactly by altering the initial model transition probability matrix via shifting probability from less costly to more costly states. How much probability can be shifted is determined by constraints 3 - 11 in formulation 4. The computational complexity is limited to the size of the transition probability matrix. Once probabilities have been found, best and worst case costs ( $v_1$  and  $v_2$  in formulation 4) will be the fixed points of the formulas in constraints 1 and 2 in formulation 4. Finding the fixed points can be done independently of finding transition probabilities and in the same manner, and with the same computational complexity, as solving non-robust asset management problems. In this way, MAXIMIN and

MAXIMAX costs and policies may be computed. The best and worst case costs are then weighted by the optimism and pessimism levels respectively and summed. Then a policy can be chosen to minimize the total costs, providing the optimal solution to the constrained Hurwicz criterion asset management problem as outlined in Formulation 4.

Hurwicz criterion based robust optimization does require the specification of both an uncertainty and an optimism level. Planning agencies may find it difficult to specify how much uncertainty they have with regards to infrastructure decay rates, or may find it undesirable to have to place a level of optimism on their management strategies. However, asset management clearly does involve managing systems with some degrees of uncertainty. The more the issue of uncertainty and the decision of how to manage it are discussed, the more informed asset management policies will be. Uncertainty levels associated with transition probability matrices can be derived from confidence intervals surrounding the statistics provided by deterioration models. Deterioration models that are based on large data sets of infrastructure deterioration over extended periods of time will produce smaller uncertainty sets than less refined models. Optimism levels are more subjective. However, since it has been shown that the asset management problem can be made robust without making its computational complexity too great, it is possible to imagine solving a particular asset management problem numerous times with various optimism (and uncertainty) levels to see how performance and reliability guarantees can be traded off.

### 3 Computational Study

In order to illustrate the application of robust dynamic programming algorithms to infrastructure management problems, an example is presented here. A one lane-mile segment of highway pavement is managed according to a policy obtained from infinite horizon robust dynamic programming. Previous research in Golabi et al (1982), Madanat (1993), and Durango and Madanat (2002) provides a ready source of data for how pavement deterioration can be modeled via static transition probabilities. However given the uncertainty in these transition probabilities, potential cost savings can be achieved by applying robust dynamic programming to this problem. It is worth noting that epistemic uncertainty is of more concern for infrastructure assets that have less refined deterioration models than pavement sections. Thus robust optimization may actually be better suited to the management of infrastructure assets like underground pipelines and drainage systems.

#### Problem Specification

When managing a section of pavement, the decisions to be made include when and how to maintain, overlay, or reconstruct the pavement. In the example presented here, it is assumed that the choices of actions to take in any given year are those presented by Durango and Madanat (2002). These actions include: (1) do nothing, (2) routine maintenance, (3) 1-in overlay, (4) 2-in overlay, (5) 4-in overlay, (6) 6-in overlay, and (7) reconstruction. The costs of the actions presented here are derived from empirical work done by Carnahan et al (1987) and are included in Table 1. These costs vary according to the condition state of the pavement. (A section of pavement is said to be in state 1 if it is unusable and in state 8 if it is brand new, with the intermediate states representing intermediate condition ratings.) Similarly, user costs associated with various pavement condition ratings are included in cal-

culations.

Table 1: Costs (dollars per lane-yard), both to users and agency, of performing different M&R actions.

	Action to take							
Condition State		1	2	3	4	5	6	7
	1	0.00	6.90	19.90	21.81	25.61	29.42	25.97
	2	0.00	2.00	10.40	12.31	16.11	19.92	25.97
	3	0.00	1.40	8.78	10.69	14.49	18.30	25.97
	4	0.00	0.83	7.15	9.06	12.86	16.67	25.97
	5	0.00	0.65	4.73	6.64	10.43	14.25	25.97
	6	0.00	0.31	2.20	4.11	7.91	11.72	25.97
	7	0.00	0.15	2.00	3.91	7.71	11.52	25.97
	8	0.00	0.04	1.90	3.81	7.61	11.42	25.97

In their study, Durango and Madanat present three sets of transition probability matrices. The matrix that describes a section of pavement deteriorating at a “medium” rate is meant to reflect the current best estimate of how a given, random section of pavement will deteriorate. This transition probability matrix is itself derived from normal distributions associated with the performance of various pavement management maintenance actions, (Madanat and Ben-Akiva 1994). The inclusion of alternative “fast” and “slow” rates of deterioration draw attention to the fact that this estimate may under or over estimate decay in meaningful ways. For the purposes of the present example, the medium decay rate transition probabilities are used to initialize the robust dynamic programming application.

Various uncertainty and optimism levels between 0 and 1 are considered. The policies obtained by robust optimization, either based on MAXIMIN or MAXIMAX or Hurwicz criteria, are compared to those obtained via non-robust optimization using the medium decay rate. Costs are then calculated in the case that the actual probabilities that guide system dynamics are somewhere between best-case and worst-case transition probabilities.

## Results

In the example case of managing pavement, robust optimal actions were found to differ substantially from actions that would be taken if the “medium” decay rate was assumed to be correct. This can be seen in Table 2, which displays MAXIMIN robust optimal policies. Note that the MAXIMIN robust policy when uncertainty level is 0 is the optimal policy for non-robust optimization.

Table 2: Maximin robust optimal actions when different uncertainty levels are considered.

Uncertainty level	Optimal action, state 8	state 7	state 6	state 5	state 4	state 3	state 2
0.0	2	3	4	4	5	6	7
0.1	2	3	4	5	6	6	7
0.2	2	3	4	5	6	6	7
0.3	2	3	4	5	6	7	7
0.4	3	4	4	5	6	7	7
0.5	4	4	4	5	6	7	7
0.6	4	4	5	6	6	7	7
0.7	4	5	6	6	6	7	7
0.8	4	5	6	6	7	7	7
0.9	4	5	6	6	7	7	7
1.0	4	5	6	6	7	7	7

The optimal management policies in MAXIMIN robust optimization are more conservative than those employed in non-robust optimization, especially as uncertainty becomes more significant. However, the actions prescribed by the MAXIMIN robust dynamic programming algorithm can yield significantly lower agency + user costs when compared to that prescribed by non-robust dynamic programming. For example, assuming transition probabilities follow those considered in the MAXIMIN robust algorithm (i.e. worst-case transition probabilities) costs are presented in Table 3.

Table 3: Worst-case future discounted costs of managing a like new pavement.

Uncertainty level	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
non-robust cost	13	30	46	62	78	94	110	126	142	154	162
maximin robust cost	13	30	46	62	72	80	80	80	80	80	80

Figure 1 depicts the cost savings achieved by using the MAXIMIN robust algorithm rather than the non-robust algorithm, both in absolute and relative terms, by uncertainty level. Note that savings from using robust optimization techniques increase both in size and in relative terms as the uncertainty level increases.

In particular, it is worth noting that the MAXIMIN approach significantly limits the range of the sum of potential user and agency costs. Figure 2 shows the ranges of potential costs of using the non-robust approach, by uncertainty level. However, the MAXIMIN robust optimization recognizes the potential for extraordinarily large costs in cases where uncertainty is high and chooses to maintain the pavement on a more regular basis in order to limit the potential maximum costs. Figure 3 shows the ranges of potential costs of using the MAXIMIN approach, by uncertainty level.

Clearly MAXIMIN asset management limits the maximum costs, but Figure 3 also shows that MAXIMIN is unable to lower costs as much as non-robust asset management systems can in the best-case situations. This is one of the shortcomings of the MAXIMIN approach, and one area in which the less conservative Hurwicz robust optimization is able to do substantially better.

Like MAXIMIN robust optimization, Hurwicz criterion based robust optimization can yield management policies that are significantly different from those provided by non-robust asset management. Optimal policies by uncertainty and optimism levels are presented in

Table 4. For each of the different uncertainty levels, optimism levels of 0.00, 0.01, ... , 1.00 were set. The table groups together ranges of the optimism level that yielded similar optimal policies.

Table 4: Hurwicz optimal actions at different uncertainty and optimism levels.

Uncertainty level	Optimism level	Optimal action in state 8	in state 7	in state 6
0.0	0.00 - 1.00	2	3	4
0.2	0.00 - 1.00	2	3	4
0.4	0.00 - 0.11	3	4	4
	0.12 - 0.41	2	4	4
	0.42 - 0.98	2	3	4
	0.99 - 0.99	2	2	4
	1.00 - 1.00	2	2	3
0.6	0.00 - 0.09	4	4	5
	0.10 - 0.14	4	4	2
	0.15 - 0.21	2	4	2
	0.22 - 0.55	2	3	2
	0.56 - 1.00	2	2	2
0.8	0.00 - 0.08	4	5	6
	0.09 - 0.12	4	5	2
	0.13 - 0.15	4	3	2
	0.16 - 0.38	2	3	2
	0.39 - 0.78	2	2	2
	0.79 - 1.00	1	2	2
1.0	0.00 - 0.07	4	5	6
	0.08 - 0.12	4	5	2
	0.13 - 0.15	4	3	2
	0.16 - 0.51	2	3	2
	0.52 - 0.81	2	2	2
	0.82 - 1.00	1	2	2

It should be noted that as uncertainty increases, the range of policies that might be optimal increases, so the choice of optimism level becomes more important. It should also be noted that, as the uncertainty level increases and the optimism level decreases, more severe M&R activities are selected. This is because it is optimal, in these cases where uncertainty is high, to follow fail-safe conservative policies. It can be seen that the non-robust optimal

policy is not chosen by the Hurwicz robust optimization for uncertainty levels greater than 0.4. This seems to indicate that the policy chosen by the non-robust optimization does a relatively poor job at least in terms of best and worst case transition probability matrices. This is logical since non-robust asset management does not consider best and worst-case scenarios, it only works with one set of transition probabilities.

Figure 4 presents the cost ranges of Hurwicz robust optimization. The Hurwicz robust optimization is able to reap the benefits of best-case transition probabilities, incurring near zero maintenance costs, but also able to limit the worst-case costs. In many ways, the cost ranges observed under this type of asset management offer a suitable compromise between the conservativeness of MAXIMIN style robust optimization and the optimism of MAXI-MAX or even non-robust DP schemes.

## 4 Conclusion

Results provided by analyzing an example problem give an indication that robust optimization has the potential to significantly reduce expected lifecycle costs in asset management in worst case conditions. If MAXIMIN formulations are deemed too conservative, alternate robust optimization methodologies like the Hurwicz criterion are available. The methodology of robust optimization provides a way to account for knowledge uncertainty within asset management systems. The robust methodology could be applied to consider uncertainty in the unit costs of M&R actions, in the funding of a planning agency, or in transition probability matrices (as in this paper). This paper demonstrates a small-scale application of a few robust optimization techniques. Alternative robust methodologies need to be investi-

gated, and robust optimization needs to be extended to multi-facility management problems.

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Figure 1: The relative and absolute benefit of using maximin robust optimization in worst-case conditions.

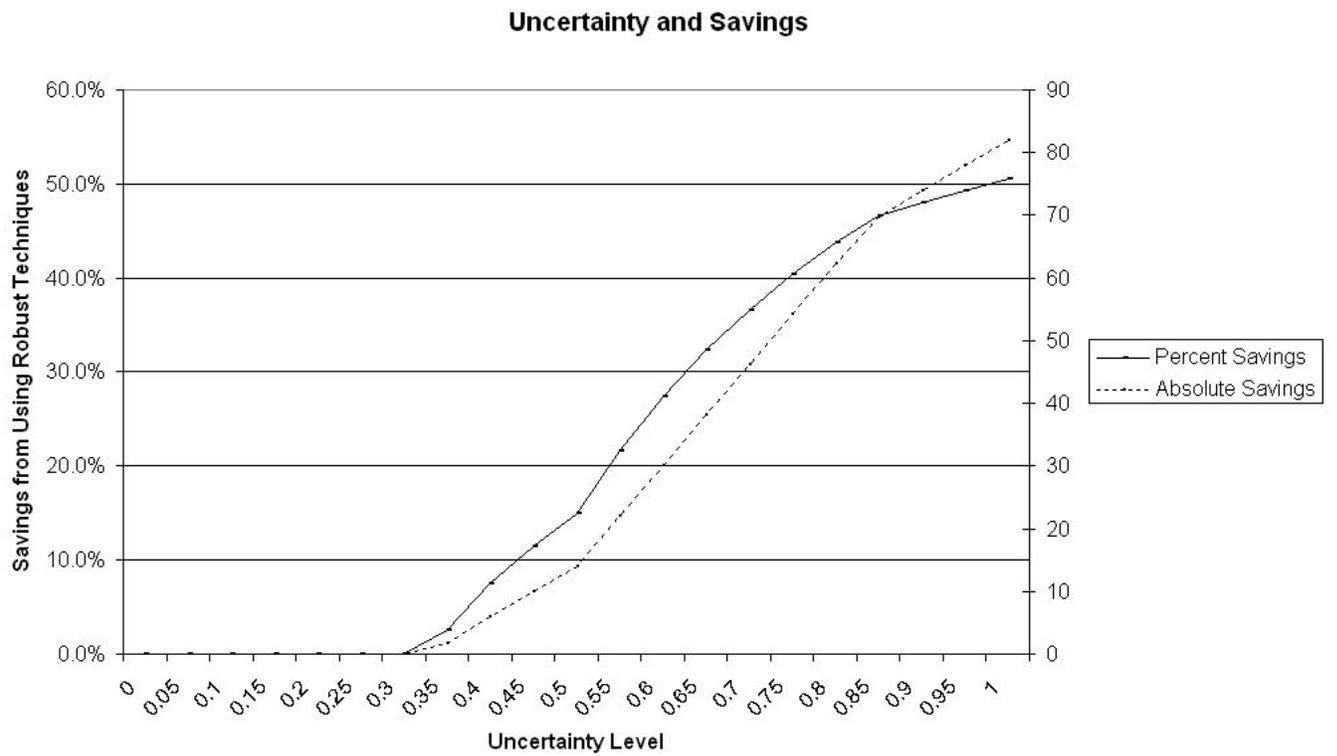


Figure 2: The cost ranges of non-robust asset management.

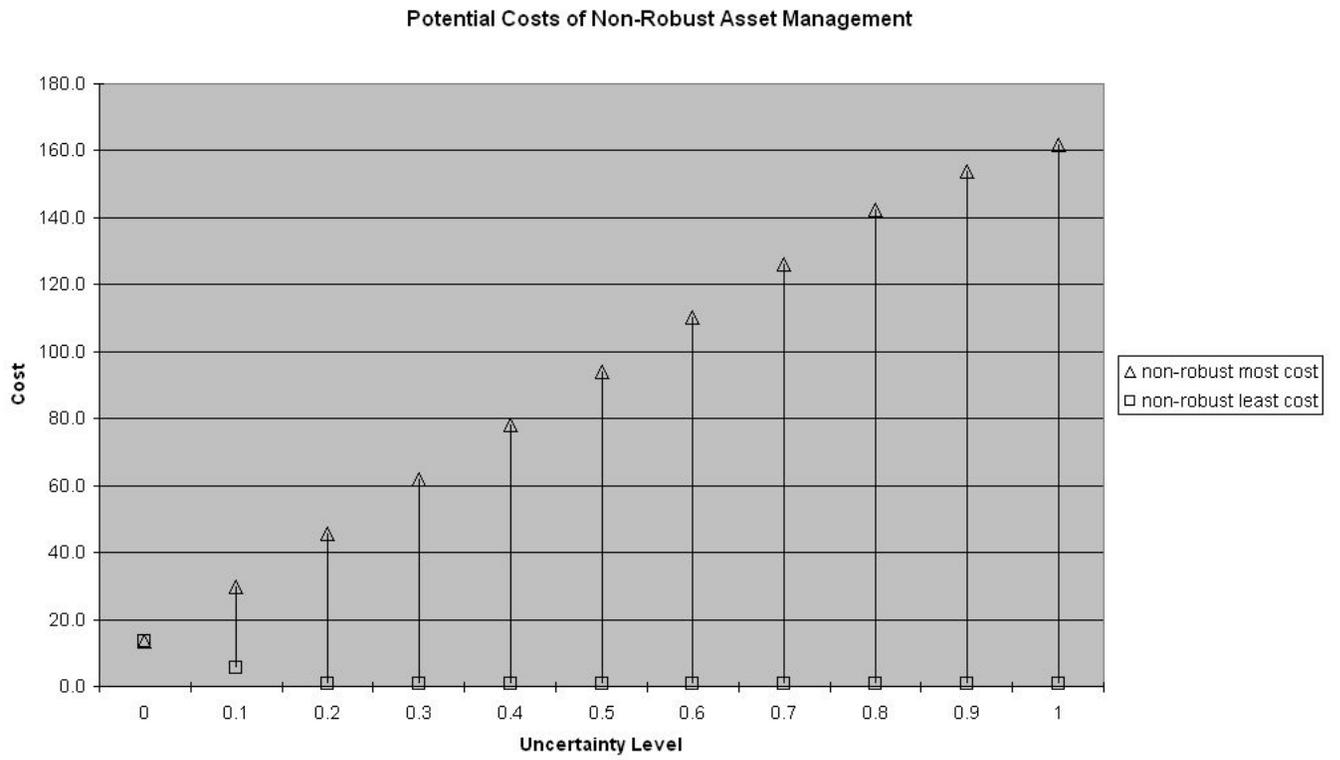


Figure 3: The cost ranges of Maximin robust asset management.

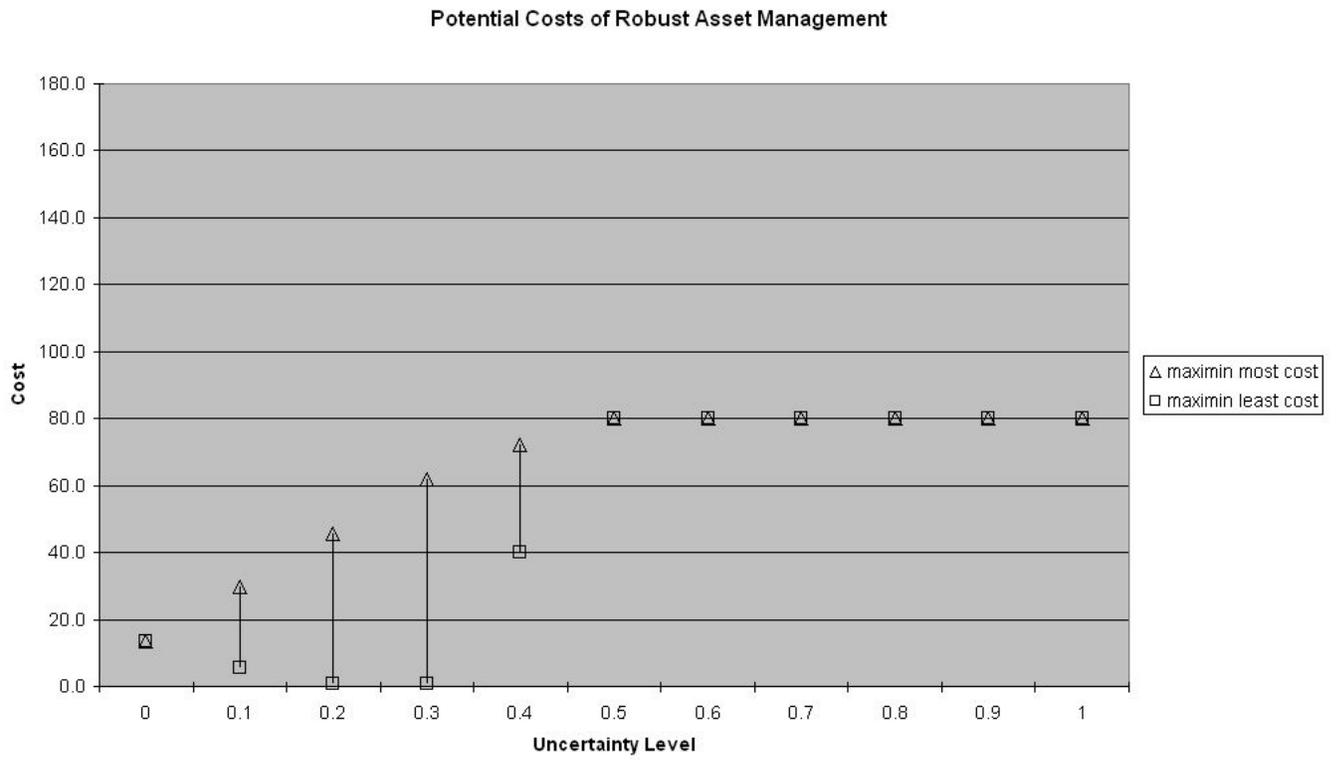


Figure 4: The cost ranges of Hurwicz (optimism = 0.5) robust asset management.

