Dynamic Programming based Maintenance and Replacement Optimization for Bridge Decks using History-Dependent Deterioration Models

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Abstract

In this research, a reliability-based optimization model of bridge maintenance and replacement decisions is developed. Bridge maintenance optimization models use deterioration models to predict the future condition of bridges. Some current optimization models use physically-based deterioration models taking into account the history of deterioration. However, due to the complexity of the deterioration models, the number of decision variables in these optimization models is limited. Some other optimization models consist of a full set of decision variables; however, they use simpler deterioration models. Namely, these deterioration models are Markovian, and the state of the Markov chain is limited to the condition of the facility.

In this research, a facility level optimization model of bridge maintenance and decisions is developed, using a Markov chain whose state includes part of the history of deterioration and maintenance. The main advantage of this formulation is that it allows the use of standard optimization techniques (dynamic programming), while using realistic, history-dependent deterioration models.

This research presents a method to formulate a realistic history-dependent model of bridge deck deterioration as a Markov chain, while retaining relevant parts of the history of deterioration, using state augmentation. This deterioration model is then used to formulate and solve a reliability-based bridge maintenance optimization problem as a Markov decision process. In a numerical example, the policies derived using the augmented Markov chain are applied to a realistic bridge deck, and compared to the policies derived using a simpler Markov chain.

Introduction

Infrastructure management is the process by which agencies monitor, maintain, and repair deteriorating systems of facilities, within the constraints of available resources. More specifically, the management process refers to the set of decisions made by an infrastructure agency over time to maximize the system performance. The basic maintenance and rehabilitation (M&R) decisions an agency has to make are: "in every time period, what M&R activity should be performed on each facility in the system?" The deteriorating bridge population, as well as the limited amount of funds available for maintenance and inspection, led to the development of bridge management systems to assist agencies to make maintenance and rehabilitation decisions by optimizing the use of available funds.

The objective of this paper is to develop a bridge component M&R optimization approach that uses a Markovian deterioration model, while accounting for aspects of the history of deterioration and maintenance. Such a model represents a compromise between simple deterioration models allowing the use of standard optimization techniques, and realistic deterioration models whose complexity prevents efficient optimization of maintenance decisions.

Review of bridge management optimization models

The optimization can be formulated as a Markov decision process (Madanat, 1993; Hawk, 1994; Golabi and Shepard, 1997; Jiang et al., 2000). In these methods, the deterioration is described by a Markov chain, with the state representing the condition of the facility. Optimal solutions are determined using dynamic programming for a single facility. The main advantage of these models is that they enable the use of standard and efficient optimization techniques. As a consequence, these models are implemented in actual Bridge Management Systems such as Bridgit and Pontis (Hawk, 1994; Golabi and Shepard, 1997). The limitation of these Markovian models is the memoryless assumption, according to which the probability for the condition of a facility to transition from an initial state A to a lower state B does not depend on the time spent in state A or on the history of deterioration and maintenance. Although parts of this assumption may be valid for certain bridge states, namely those where the deterioration is primarily governed by mechanical processes, Mishalani and Madanat (2002) have shown empirically that it is unrealistic for bridge states where the deterioration is primarily governed by chemical processes.

Deterioration models in which the history of deterioration is taken into account exist and have been used in bridge maintenance optimization (Mori and Ellingwood, 1994; Kong and Frangopol, 2003; Robelin and Madanat, 2006). However, due to the complexity of their underlying deterioration models, these optimization methods use a very limited number of decision variables in order to remain tractable. To the knowledge of the authors, there does not exist a bridge maintenance optimization method that has more than a few decision variables and that is based on a deterioration model that takes into account the history of deterioration and maintenance. The purpose of the present article is to develop a bridge maintenance optimization method with a more complete set of decision variables, while using a deterioration model that takes into account important aspects of the history of deterioration and maintenance.

Formulation of a history-dependent deterioration model as a Markov model

Objective. The objective of the present section is to develop a model of the deterioration of a bridge deck with the two following characteristics: the model is Markovian, and it takes into account aspects of the history of deterioration and maintenance.

Definitions and assumptions. The system considered is a single bridge deck managed by an agency such as a state Department of Transportation. Maintenance decisions are made by the agency at discrete points in time, every year. The condition of the deck is represented by its condition index β. By definition of the condition index, the instantaneous probability of failure of the deck (given it has not failed yet) is $\Phi(-\beta)$, where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.

State of the Markov chain. In earlier Markovian deterioration models, the state is an integer representing the condition of the deck. In the present model, the condition index β of the deck is also part of the state of the Markov chain, as well as three additional variables:
• β^0: the condition index of the deck after the latest maintenance action was performed, or when the deck was new if no maintenance action has been performed yet,
• m: an integer indicating the type of the latest maintenance action performed on the deck (or 0 if no maintenance action has been performed since the deck was new), and
• τ: the time since the latest maintenance action (or the time since the deck was new, if no maintenance action has been performed yet).

The state $x = (\beta, \beta^0, m, \tau)$ consists of two real numbers in the general case (β and β^0) and two integers (m and τ), since there is a finite number of different types of maintenance actions and the unit of time is the year. In practice, the set of possible values for each variable can actually be restricted to small intervals while maintaining the full functionality of the model.

Estimation of the transition probabilities. Transition probabilities represent the probability for a facility that is in state $x_t = (\beta_t, \beta_t^0, m_t, \tau_t)$ at time step t to be in state $x_{t+1} = (\beta_{t+1}, \beta_{t+1}^0, m_{t+1}, \tau_{t+1})$ at the following time step. This transition probability is denoted as $P\left(x_{t+1} \middle| x_t\right)$. Note that x_t and x_{t+1} can be any elements of the state space, and may or may not be equal. The original deterioration model of the facility, which is stochastic, is used to estimate the transition probabilities for the resulting Markovian model. In order to accommodate any original deterioration model, Monte Carlo simulation is used to estimate the transition probabilities.

Formulation of the optimization as a Markov Decision Process

Definitions and assumptions. As described earlier, the system considered is a bridge deck managed by an agency. The agency incurs costs when maintenance actions are performed or when the deck is replaced. Moreover, maintenance actions on a deck or its replacement imply the closure of some or all of its lanes. This leads to delays to the users and/or costs associated with detours.

The agency is responsible for the maintenance and replacement of the bridge for the duration of the planning horizon (T years), after which the bridge is assumed to have a salvage value of V^S.

Problem formulation. Since the deterioration model developed earlier is Markovian, the optimization problem can be formulated as a Markov decision process (Bertsekas, 2001). The following notation is used:
• X: state space of the Markov chain representing the deterioration of the deck. X is the set of all possible values for $(\beta, \beta^0, m, \tau)$, as defined in the previous section.
• U: set of all possible M&R actions, i.e. all types of maintenance actions, replacement, or do-nothing.
• c_u: cost of action u.
• α: discount factor; $\alpha = 1/(1+r)$, where r is the discount rate.
• $V_t(x)$: minimum cost-to-go for the agency to manage the bridge deck from year t to the end of the planning horizon, starting from state x in year t.
• μ: set of optimal decisions. $\mu_t(x)$ is the optimal decision when the bridge deck is in state x in year t. It is the result of the optimization.

The problem formulation is as follows.

$$\begin{array}{rcll} \forall x \in X, & & & \\ V_t(x) & = & \min_{u \in U} \left\{ c_u + \alpha \sum_{y \in X} P(y|x) V_{t+1}(y) \right\} & \text{if } t \in \{0, \cdots, T-2\} \\ & = & V^s & \text{if } t = T-1 \\ \text{subject to} & & & \\ \Phi(-\beta_t) & \leq & P^{\text{acceptable}}, & t \in \{0, \cdots, T\} \end{array} \quad (1)$$

where y is a summation variable representing each state of the Markov chain, and $P^{\text{acceptable}}$ is a user-defined upper bound on the probability of failure.

Solution. The problem formulated above can be solved using backward recursion (Bertsekas, 2001). The minimum discounted cost to manage the bridge over the whole planning horizon is $V_0(x_0)$, where x_0 is the initial state of the bridge.

Case study

This section compares the policies derived using the augmented state Markovian model proposed in this article and the policies derived using a simpler Markovian model, in which the state is the condition of the facility in the current year.

For each Markovian model:
• the coefficients of the transition probability matrices are estimated by Monte-Carlo simulation, as explained earlier, using deterioration parameters adapted from Frangopol et al. (2001),

• the set of policies is determined, using dynamic programming as explained earlier. For this, the costs of maintenance and replacement are adapted from Kong and Frangopol (2003).

Let us call "policies of the simple model" the policies obtained using the simple Markovian model, and "policies of the augmented state model" the policies obtained using the Markovian model with state augmentation. The application of these two sets of policies is finally simulated on two bridge decks having the same deterioration parameters, over a time horizon of 75 years. In the example described in this article, the total cost using the policies of the simple model is 63 percent more than the total cost using the policies of the augmented state model (4,215 dollars per square meter of deck area when using the simple model, and 2,580 dollars per square meter of deck area when using the augmented state model). As shown in Figure 1, the condition of the deck is worse on average when using the policies of the simple model. Moreover, several different values for the deterioration and cost parameters were tried, and the savings obtained through the use of the augmented state model were always significant. The sequence of M&R actions obtained by application of the policies of the simple model is very different from the sequence obtained by application of the policies of the augmented state model, as shown in Figure 1.

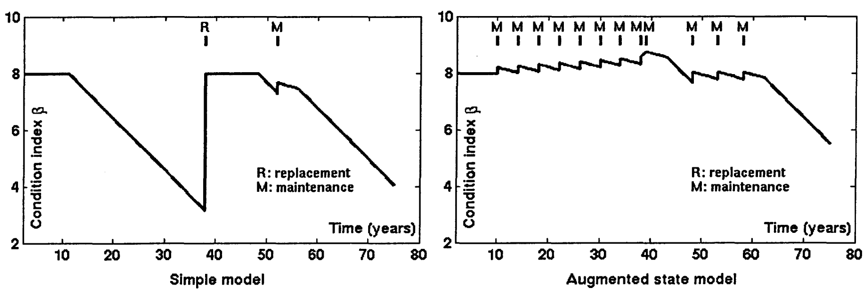

Figure 1: Evolution of the bridge deck condition over the planning horizon, for two scenarios: application of the policies of the simple model (left), and application of the policies of the augmented state model (right).

Using the policies of the augmented state model, the performance of maintenance actions at almost regular intervals is a result of the optimization, and was not provided as an input to the model. A possible intuitive explanation for this fact is as follows. By construction, the state space of the augmented state model captures more detail than the state space of the simple model. In particular, the combination of values for the condition of the facility and for the time since the previous maintenance action is possible in the augmented state model, and not in the simple model. This combination allows for more selective recommendations using the augmented state. For example, if the current condition is 5, the recommendation using the augmented state model may be to perform maintenance if the previous maintenance action was performed seven years before or earlier, and to do nothing if the previous maintenance action was performed less than seven years before. In the same situation, if the current condition is

5, the simple model provides only one recommendation, regardless of the time since the previous action. Thus, the performance of maintenance at regular intervals cannot be recommended by the simple model.

Conclusion

This research presents a method to formulate a realistic history-dependent model of bridge deck deterioration as a Markov chain, while retaining relevant parts of the history of deterioration, using state augmentation. This deterioration model is then used to formulate and solve a reliability-based bridge maintenance optimization problem as a Markov decision process. The numerical example demonstrates the savings brought by the application of the policies of the augmented state model compared to a simpler Markovian model, therefore showing that the additional information included in the model is indeed beneficial.

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