History-Dependent Bridge Deck Maintenance and Replacement Optimization with Markov Decision Processes

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Abstract: Bridge maintenance and replacement optimization methods use deterioration models to predict the future condition of bridge components. The purpose of this paper is to develop a framework for bridge maintenance optimization using a deterioration model that takes into account aspects of the history of the bridge condition and maintenance, while allowing the use of efficient optimization techniques. Markovian models are widely used to represent bridge component deterioration. In existing Markovian models, the state is the bridge component condition, and the history of the condition is not taken into account, which is seen as a limitation. This paper describes a method to formulate a realistic history-dependent model of bridge deck deterioration as a Markov chain, while retaining aspects of the history of deterioration and maintenance as part of the model. This model is then used to formulate and solve a reliability-based bridge maintenance optimization problem as a Markov decision process. A parametric study is conducted to compare the policies obtained in this research with policies derived using a simpler Markovian model.

Introduction

Based on the recent status of United States' highways, bridges, and transit (FHwA 2002), the average year of construction of the bridges in the United States was determined to be 1963. In 2002, 50% of the national daily traffic were utilizing bridges that were older than forty years. Of the 586,000 bridges in the nation, 28% were found deficient, half of which were structurally deficient. The deteriorating bridge population, as well as the limited amount of funds available for maintenance and inspection, led to the development of bridge management systems to assist agencies to make maintenance and rehabilitation decisions by optimizing the use of available funds.

Experience with infrastructure management systems in the United States shows that the benefits of systematic approaches to facilities management have been substantial in practice. For example, the Arizona Department of Transportation reported that the implementation of their pavement management system (PMS) to optimize pavement rehabilitation expenditures has saved over US$200 million in maintenance and rehabilitation costs over a five-year period (OECD 1987). These savings were achieved because the maintenance and rehabilitation resource allocation decisions were made by the PMS with an objective to minimize the life cycle costs of the pavement sections in the network. In the area of bridges, the Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 required every state department of transportation and metropolitan planning organization to implement a bridge management system in order to optimize the allocation of resources for maintenance planning. This federal mandate has recently been waived (Schweppe 2001). Despite the waiver, government agencies are still faced with aging infrastructure and limited resources. Thus, there is a definite need for the optimization of the allocation of resources for maintenance planning.

The objective of this paper is to develop a bridge component maintenance and replacement (M&R) optimization approach that uses a Markovian deterioration model, while accounting for aspects of the history of deterioration and maintenance. Such a model represents a compromise between simple deterioration models allowing the use of standard optimization techniques, and realistic deterioration models whose complexity prevents efficient optimization of maintenance and replacement decisions.

The article is organized as follows. The next section reviews some of the existing bridge maintenance optimization methods, with emphasis on the relationship between complexity of the underlying model and usefulness of the optimization method. The subsequent section describes how a realistic and complex deterioration model can be formulated as a Markov chain, while maintaining some of the characteristics of the original model. The optimization of maintenance and replacement decisions is then formulated, using the deterioration model developed earlier. A numerical study is conducted in the last section in order to compare the policies obtained using the model and method developed

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in this research with policies derived using a simpler Markovian model.

Review of Bridge Management Optimization Methods

Given available resources and a set of possible M&R actions, the objective of infrastructure management is to determine the optimal M&R decisions in the current year and in future years. The solution is based on the consequences of possible actions on the future condition of the system. Since information about the future condition is not available, deterioration models are used. This framework is common to all existing bridge management optimization methods, although the actual formulation of the optimization and the deterioration models differ.

The optimization can be formulated as a Markov decision process (Madanat 1993; Hawk 1994; Golabi and Shepard 1997; Jiang et al. 2000). In these models, the deterioration is described by a Markov chain, with the state representing the condition of the facility. Optimal solutions are determined using dynamic programming for a single facility or linear programming for a system of several facilities. The main advantage of these models is that they enable the use of standard and efficient optimization techniques. As a consequence, these models are implemented in actual bridge management systems such as Bridgit and Pontis (Hawk 1994; Golabi and Shepard 1997), and also serve as a basis for more refined models using adaptive control (Durango and Madanat 2002) or taking into account inspection errors (Madanat 1993; Jiang et al. 2000). The limitation of these Markovian models is the memoryless assumption, according to which the probability for the condition of a facility to transition from an initial State A to a lower State B does not depend on the time spent in State A or on the history of deterioration and maintenance. Although parts of this assumption may be valid for certain bridge states, namely, those where the deterioration is primarily governed by mechanical processes, Mishalani and Madanat (2002) have shown empirically that it is unrealistic for bridge states where the deterioration is primarily governed by chemical processes.

Deterioration models in which the history of deterioration is taken into account exist and have been used in bridge maintenance optimization (Mori and Ellingwood 1994; Kong and Frangopol 2003; Robelin and Madanat 2006). However, due to the complexity of the deterioration models, these optimization methods can only handle a very limited number of decision variables, since the computation of the optimal solution requires the enumeration of the consequences of all possible actions over all stage–state combinations. To the knowledge of the writers, there does not exist a bridge maintenance optimization method that has more than a few decision variables and is based on a deterioration model that takes into account the history of deterioration and maintenance.

The purpose of the present paper is to develop a bridge deck maintenance and replacement optimization method with a more complete set of decision variables, while using a deterioration model that takes into account important aspects of the history of deterioration and maintenance. These aspects include the time since the performance of the latest maintenance action and the type of that activity.

Formulation of a History-Dependent Deterioration Model as a Markov Model

The objective of the present section is to develop a model of the deterioration of a bridge deck with the two following characteristics: the model is Markovian and it takes into account aspects of the history of deterioration and maintenance.

Definitions and Assumptions

The system considered is a single bridge deck managed by an agency such as a state department of transportation. Maintenance and replacement decisions are made by the agency at discrete points in time. Due to the presence of yearly budget constraints, the most logical time unit for M&R decisions is the year. As a consequence, the model of deterioration is also discretized in time intervals of one year. This assumption is consistent with the typical rate of deterioration of bridges. Within a year, the actual point in time at which the action (maintenance or replacement) is performed is not important, and an action is said to be performed "in year n."

The condition of the deck is represented by its reliability index β. By definition of the reliability index, the instantaneous probability of failure of the deck (given it has not failed yet) is $\Phi(\beta)$, where $\Phi(\cdot)=$ standard normal cumulative distribution.

Several types of maintenance actions can be performed on the bridge deck. Maintenance actions of different types have different influences on the condition of the deck, as well as different costs. The deck can also be replaced, in which case it is then considered new.

Methodology

Definition of State of the Markov Chain

In earlier Markovian deterioration models, the state is an integer representing the condition of the deck. In the present model, the reliability index β of the deck is part of the state space of the Markov chain, as well as additional variables that are known to the decision maker and can provide an advantage in selecting the optimal actions. This process is known as state augmentation (Bertsekas 2001). The variables added to the state space are:
- $m=$ integer indicating the type of the latest action (maintenance or replacement) performed on the deck (or 0 if no action has been performed since the deck was new); and
- $\tau=$ time since the latest action (or the time since the deck was new, if no action has been performed yet).

The choice of these variables is based on the precision and ease of their measurement, as well as on their contribution to the reduction of the uncertainty in modeling the deterioration. The number of additional variables must also remain reasonably low, so that the model can be implemented. These two variables have been frequently used in infrastructure performance modeling (Ben-Akiva and Gopinath 1995; Mishalani and Madanat 2002).

The state $x=(\beta,m,\tau)$ consists of one real number in the general case (β) and two integers (m and τ), since there is a finite number of different types of maintenance actions and the unit of time is the year. In practice, the set of possible values for each variable can actually be restricted to small intervals while maintaining the full functionality of the model. For example, typical values of β are integers between 1 and 15.

Table 1. Pseudocode: Estimation of Transition Probabilities Using Monte Carlo Simulation

1	Define parameters: `BETA`, (set of possible values for β - current condition - and β^{ini} - condition after previous action), `M` (set of possible values for m), `TAU` (maximum value of τ), `N` (number of Monte Carlo trials)
2	Initialize `Ntotal[beta, m, tau]` at 0 for all values of beta in `BETA`, m in `M`, and tau in `TAU`
3	Initialize `Ntr[beta1, m1, tau1, beta2, m2, tau2]` at 0 for all values of `beta1` and `beta2` in `BETA`, `m1` and `m2` in `M`, and `tau1` and `tau2` in `TAU`
4	Repeat `N` times:
5	Repeat for all values of `m` in `M`:
6	Repeat for all values of `betaIni` in `BETA`:
7	Draw an instance of a set of deterioration parameters from their known distributions
8	Repeat for `tau` from 0 to `TAU`:
9	Determine the (continuous) condition `betaCont [tau]`
10	Determine the corresponding discretized condition `beta [tau]`, where `beta [tau]` is in the set `BETA`
11	End repeat
12	Repeat for `tau` from 0 to `TAU-1`:
13	Increment `Ntotal [beta [tau], m, tau]` by 1
14	Increment `Ntr [beta [tau], m, tau, beta [tau+1],m, tau+1]` by 1
15	End repeat
16	End repeat
17	End repeat
18	End repeat
19	`Ntr [beta1, m1, tau1, beta2, m2, tau2]` divided by `Ntotal [beta1, m1, tau1]` is an estimate of the probability of transition from state (`beta1, m1, tau1`) to state (`beta2, m2, tau2`)

Estimation of the Transition Probabilities

Transition probabilities represent the probability for a facility that is in state $x_t = (\beta_t, m_t, \tau_t)$ at time period t to be in state $x_{t+1} = (\beta_{t+1}, m_{t+1}, \tau_{t+1})$ at the following time period. This transition probability will be denoted as

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t) \qquad (1)$$

Note that x_t and x_{t+1} can be any elements of the state space, and may or may not be equal. The original deterioration model of the facility, which is stochastic, is used to estimate the transition probabilities for the resulting Markovian model. In order to accommodate any original deterioration model, Monte Carlo simulation is used to estimate the transition probabilities. Namely, a large number of deterioration profiles are generated using the original deterioration model, and the counts shown on the right-hand side of Eq. (2) are determined

$$P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t) = \frac{N^{\text{transitions}}_{(\beta_t, m_t, \tau_t),(\beta_{t+1}, m_{t+1}, \tau_{t+1})}}{N^{\text{total}}_{(\beta_t, m_t, \tau_t)}} \qquad (2)$$

where
- $N^{\text{total}}_{(\beta_t, m_t, \tau_t)}$ = number of occurrences of the following situation, among all time steps of all Monte Carlo trials: "the deck is in state (β_t, m_t, τ_t)"; and
- $N^{\text{transitions}}_{(\beta_t, m_t, \tau_t),(\beta_{t+1}, m_{t+1}, \tau_{t+1})}$ = number of occurrences of the following situation, among all time steps of all Monte Carlo trials: "the deck is in state (β_t, m_t, τ_t) at a time step and in state $(\beta_{t+1}, m_{t+1}, \tau_{t+1})$ at the following time step."

The pseudocode for the estimation of the transition probabilities using Monte Carlo simulation is shown in Table 1. Several simplifications are possible in order to decrease the computation time necessary to estimate all transition probabilities. For example, for any β_t and β_{t+1}, $P(\beta_{t+1}, m_{t+1}, \tau_{t+1} | \beta_t, m_t, \tau_t)$ is necessarily zero if $m_{t+1} = m_t$ and $\tau_{t+1} \neq \tau_t + 1$, or if $m_{t+1} \neq m_t$ and $\tau_{t+1} \neq 0$. The pseudocode is presented for the particular example of the deterioration model from Frangopol et al. (2001). If the deck condition were measured by its serviceability, empirical condition data

would be available, and the methodology presented in this paper could also be applied to determine transition probabilities.

Formulation of the Optimization as a Markov Decision Process

The model of deterioration of a bridge deck developed in the previous section can be used in the optimization of maintenance and replacement decisions for that deck.

Definitions and Assumptions

As described earlier, the system considered is a bridge deck managed by an agency. The agency incurs costs when maintenance actions are performed or when the deck is replaced. Moreover, maintenance actions on a bridge or deck replacement usually imply the closure of some or all of its lanes. This leads to delays to the users or costs associated with detours. This is particularly important in the case of bridges. In a highway network, bridges are usually capacity constraining, due to their high cost of construction relative to regular highway lanes. Moreover, convenient detours may not be available. In pavement management, the influence of the facility condition on the users is usually modeled by user costs. The user costs represent vehicle wear and tear, and fuel consumption due to rough roads and a translation in monetary values of the riding discomfort. In the case of bridges, users are assumed indifferent to the facility condition, as long as there is no failure. Bridges are usually short compared to the total distance of a trip, and the roughness of their surface does not influence significantly the total fuel consumption and the overall driving comfort. Thus, user costs consist of delays, closures, and detours associated with the performance of maintenance actions and deck replacement.

The agency is responsible for the maintenance and replacement of the bridge for the duration of the planning horizon (T

Table 2. Pseudocode: Backward Recursion Algorithm

1	Define parameters (similar notations as defined in the problem formulation and in the pseudocode for the Monte Carlo simulation used to estimate the transition probabilities). In particular, define the threshold of reliability index, and the cost of actions `c [ u, beta]` accordingly.
2	Initialize `V [T-1, x]` to VS for all possible values of `x=(beta,m,tau)`
3	Repeat for `t` from `T-2` to `0` (step-1):
4	Repeat for all possible values of `x=(beta,m,tau)`:
5	`Vtmp ← ∝`
6	Repeat for all possible actions `u`:
7	If `c[u,beta]+alpha`$^*\Sigma_{y\in X}$`P(y|x,u)`*`V[t+1,y]<Vtmp`
8	`Vtmp ← c[u,beta]+alpha`$^*\Sigma_{y\in X}$`P(y|x,u)`*`V[t+1,y]<Vtmp`
9	`muTmp ← u`
10	End if
11	End repeat
12	`V[t,x] ← Vtmp`
13	`mu[t,x] ← muTmp`
14	End repeat
15	End repeat

years). The planning horizon is assumed to start at year 0, and is broken down into periods of one year. The agency makes maintenance and replacement decisions every year.

It is assumed that there is perfect information regarding the past and present reliability index of the bridge deck. From a practical point of view, this means that inspections are carried out frequently, such as on a yearly basis, and that the inspections are error free.

Problem Formulation

Since the deterioration model developed earlier is Markovian, the optimization problem can be formulated as a Markov decision process (Bertsekas 2001). The following notation is used:

- X=state space of the Markov chain representing the deterioration of the deck. X=set of all possible values for (β, m, τ), as defined in the previous section;
- U=set of all possible M&R actions, i.e., all types of maintenance actions, replacement, or do nothing;
- $c_u(\beta)$=cost of action u performed on bridge deck in condition β. For state $x=(\beta, m, \tau)$, the notation $c_u(x)$ is also used, and refers to $c_u(\beta)$;
- α=discount factor, $\alpha=1/(1+r)$ where r=discount rate;
- $V_t(x)$=minimum cost-to-go for the agency to manage the bridge deck from year t to the end of the planning horizon, starting from state x in year t; and
- μ=set of optimal decisions. $\mu_t(x)$=optimal decision when the bridge deck is in state x in year t.

The problem formulation is as follows:

$$\forall x \in X, V_t(x) = \min_{u \in U}\left\{ c_u(x) + \alpha \sum_{y \in X} P(y|x)V_{t+1}(x) \right\} \text{ if }$$

$$t \in \{0, \ldots, T-2\} = V^S \quad \text{if } t = T-1 \tag{3}$$

where V^S captures the usefulness of the bridge deck past the planning horizon. For practical purposes, the influence of the actual value of V^S is limited if the discount rate r is strictly positive.

Solution

The problem formulated above can be solved using backward recursion, as shown in Table 2. The minimum discounted cost to

manage the bridge deck over the whole planning horizon is $V_0(x_0)$, where x_0=initial state of the bridge deck. Using the notation of the pseudocode, the minimum discounted cost is $V(0, x_0)$.

Numerical Study

Comparison with Simpler Markovian Models

The objective of this section is to compare the policies derived using the augmented state Markovian model proposed in the present paper and the policies derived using a simpler Markovian model. Let us refer to the augmented state Markovian model as the three-dimensional (3D) model. The state of the simpler Markovian model used as a comparison is composed of one variable only, the current condition of the deck. This model is referred to as the one-dimensional (1D) model. For each Markovian model:

- The coefficients of the transition probability matrices are estimated using Monte Carlo simulation, as explained above, using the same deterioration parameters adapted from Frangopol et al. (2001); and
- The set of policies is determined using dynamic programming as explained above. For this, the costs of maintenance and replacement, including user costs, are adapted from Kong and Frangopol (2003).

The application of the sets of policies derived using the 1D and 3D models is finally simulated on two bridge decks having the same deterioration parameters, over a time horizon of 75 years (Fig. 1).

In the simulation, the deterioration of the bridge deck is determined by a model such as the one presented in Frangopol et al. (2001), and not using the transition probabilities derived earlier. Therefore, the comparison is made on the basis of a model assumed to be an accurate representation of reality, and more importantly, independent from the method used to determine the policies. Specifically, the comparison would be much weaker if it were based on the values of the objective functions (predicted optimal costs and not simulated costs).

Comparison of the 3D and 1D Models for One Facility

The applications of the policies determined using the 3D and 1D models are first compared for one facility. Fig. 2 shows the evo-
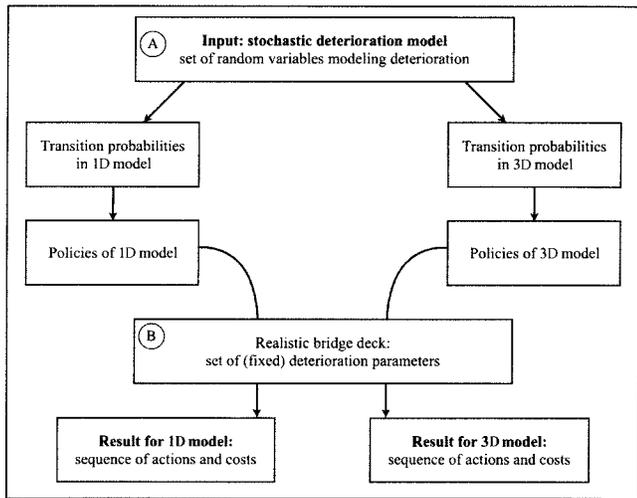
Fig. 1. Comparison of two optimization methods (1D and 3D models)

Fig. 2. Comparison of the application of the policies of the (A) 1D model; (B) 3D model

lution of the condition of a bridge deck if the policies of the 1D model are applied (top graph), and if the policies of the 3D model are applied (bottom graph). Fig. 2 also shows the sequence of actions performed in each case.

The main objective of this comparison is to show the difference in the sequence of actions when applying the two sets of policies. Cost comparisons are presented in the probabilistic sense in the next section. The sequence of M&R actions obtained by application of the policies of the 1D model is very different from the sequence obtained by application of the policies of the 3D model. Namely, using policies of the 1D model, the deck is replaced twice, at years 32 and 64, and maintenance is never performed. Using the policies of the 3D model, maintenance is performed regularly, every three to four years. Maintenance is not performed at the end of the planning horizon, since the final condition of the deck is indifferent, provided it is above the user-defined threshold of reliability index. The deck is not replaced over the planning horizon.

Using the policies of the 3D model, the performance of maintenance actions at almost regular intervals is a result of the optimization and was not provided as an input to the model. A possible intuitive explanation for this fact is as follows. By construction, the state space of the 3D model captures more detail than the state space of the 1D model. In particular, the combination of values for the condition of the facility and for the time since the previous maintenance action is possible in the augmented-state model and not in the 1D model. This combination allows for more selective recommendations using the 3D model. For example, if the current condition is 5, the recommendation using the 3D model may be to perform maintenance if the previous maintenance action was performed five years before or earlier, and to do nothing if the previous maintenance action was performed less than five years before. In the same situation, if the current condition is 5, the simple model provides only one recommendation, regardless of the time since the previous action. Thus, the performance of maintenance at regular intervals cannot be recommended by the 1D model.

This behavior is not limited to the example presented in this section. The application of the policies with many other deterioration parameters produced the same pattern of sequences of actions. More precisely, a large number of deterioration parameters
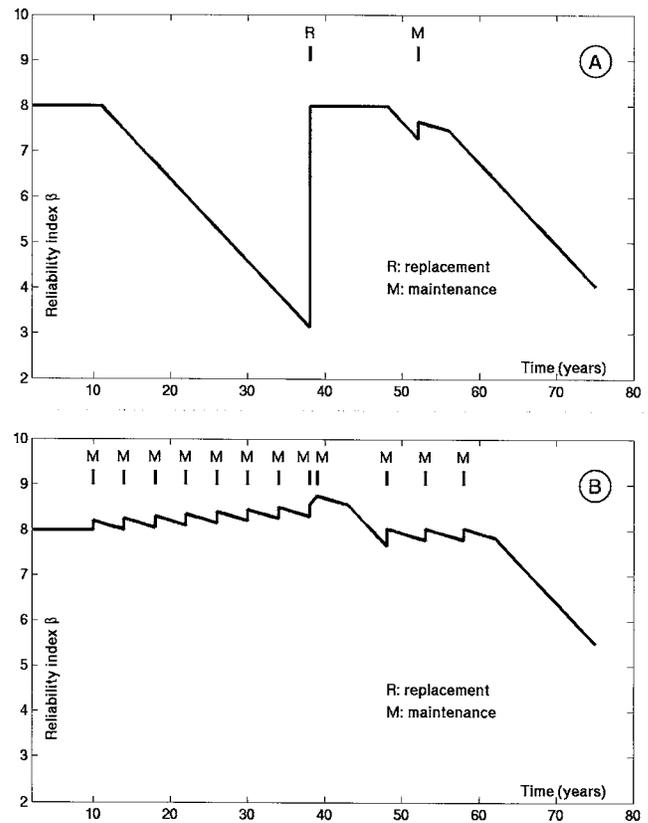
were drawn from the distributions presented in Frangopol et al. (2001), thus creating a large number of test bridge decks, and the pattern of the sequences of actions described in this section occurred in the majority of the cases.

Comparison of the 3D and 1D Models in the Probabilistic Sense

In the previous paragraph, the comparison of the two models was mostly qualitative, in terms of the general patterns of the sequence of actions. The purpose of the present comparison is to analyze the difference in costs resulting from the application of the policies of the 3D and 1D models, based on trials on a large number of test bridge decks representing a realistic range of deterioration parameters.

The input deterioration model, as described in Box A in Fig. 1, remains the same, which implies that the policies remain the same as well, for each model (and for a value of the user-defined threshold of reliability index). These two sets of policies are applied to a variety of different bridge decks (Box B in Fig. 1). To create a realistic set of different facilities, the deterioration parameters for each bridge are drawn from the distributions provided in Frangopol et al. (2001). The distributions used as input to determine the optimal policies and the distributions used to draw the population of test facilities are the same. This is not a coincidence, but rather, a consequence of the following assumption: the stochastic deterioration model is assumed to be known and is assumed to be the same for each test bridge deck.

Table 3. Pseudocode: Determination of Optimal Policies and Simulation of the Application of the Policies to a Population of Bridges, for One Model

1	Define distributions **D** of random variables modeling the deterioration (Box A in Fig. 1)
2	Determine the transition probabilities (pseudocode provided earlier)
3	Define parameters: `N` (number of trials), `B` (set of user-defined thresholds of reliability index)
4	Repeat for all values of `b` in `B`:
5	Determine the optimal policies **P(b)**, with `b` as the user-defined threshold of reliability index (pseudocode provided earlier)
6	End repeat
7	Initialize `total cost [b, i]` at 0 for all values of `b` in `B` and for `i` from `1` to `N`
8	Repeat for all values of `b` in `B`:
9	Repeat for `i` from `1` to `N`:
10	Draw an instance of a set of deterioration parameters (Box B in Fig. 1) from the distributions **D**
11	Simulate the evolution of the condition of the facility, applying actions according to policies **P(b)** (recording yearly costs)
12	Record simulated total cost in `total cost [b, i]`
13	End repeat
14	End repeat

Comparison of Average Costs

The average total cost over the planning horizon when applying the policies of the 3D model is determined for a large number of different test bridge decks. This average is also determined in the case of the policies of the 1D model. Moreover, these simulations are done for several different values of the user-defined threshold of reliability index. The pseudocode for the determination of the policies and the simulations is presented in Table 3.

This simulation is done for the 3D and 1D models. As shown by the results in Fig. 3, the average cost when using the policies of the 3D model is approximately 30% lower than the average cost when the policies of the 1D model, based on 100,000 trials. It can be noted that, for a given model, the mean simulated cost increases as the threshold of reliability index increases (i.e., as the facility becomes more reliable), which is intuitive.

Probability of Lower Cost

Using the simulation described earlier, it is possible to obtain the empirical distribution of the simulated costs. This, in turn, allows for the determination of the probability of a lower cost when applying the policies of the 3D model than when applying the policies of the 1d model. The empirical distribution of the simulated costs is shown in Fig. 4. Using these joint distributions, it can be determined that the probability of a lower cost when applying the policies of the 3D model than when applying the poli-

cies of the 1D model is 0.75. Moreover, a Kolmogorov–Smirnov test run on the sample costs allows to reject the following hypothesis: "the sample costs when applying the policies of the 3D model and the sample costs when applying the policies of the 1D model are drawn from the same distribution." This formally proves that the difference in the distribution of the costs intuitively apparent in Fig. 4 is indeed statistically significant.

Computational Considerations

The computation times needed for the implementation of the 3D model are very short on a computer with a 1.7 GHz processor, 1 GB RAM, running Unix. The computation time for the estimation of the transition probabilities is approximately one minute, with 100,000 Monte Carlo trials. The computation time for the determination of optimal policies is of the order of a few seconds, with a state space of size 840 and a time horizon of 75 years. The computation times needed for the implementation of the 1D model are even shorter. In the 3D model, the state space consists of 14 different values for the current condition, two types of actions (maintenance and replacement), and 30 different values for the time since the previous action. In the 1D model, the state space is of size 14, which is the number of different values for the current condition. The limitation for the state space is actually not the computation time, but the memory needed to implement the backward recursion algorithm.
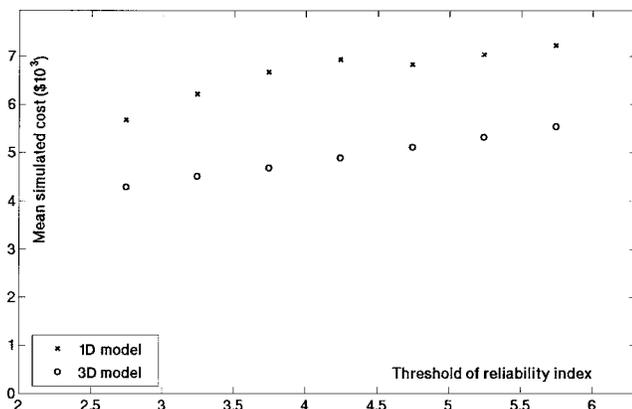
Fig. 3. Comparison of the mean simulated total costs when applying the policies of 3D and 1D models for different values of threshold of reliability index
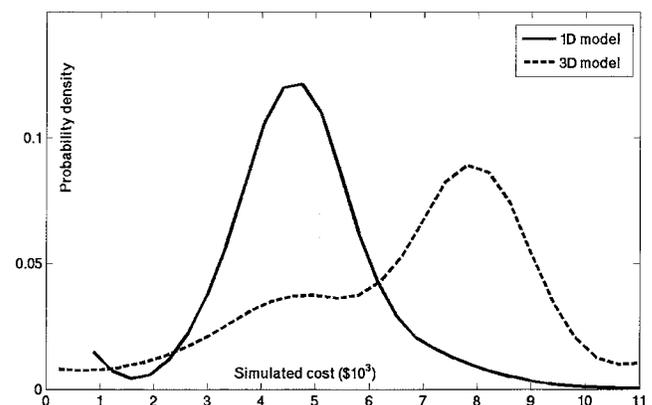
Fig. 4. Comparison of the distribution of simulated costs when applying the policies of the 3D and 1D models

Conclusion

We have presented an approach to formulate a complex history-dependent deterioration model as a Markovian model with augmented state, as well as its use in a Markov decision process to determine optimal maintenance and replacement policies for one facility. Additional research is needed to address the problem of determining optimal maintenance and replacement policies for a system of facilities. Given the short computation times, the method presented in this article for facility level optimization seems to be a promising candidate to be adapted to the optimization at the system level.

Acknowledgments

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