A Bid Analysis Model with Business Constraints for Transportation Procurement Auctions

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Abstract

Business to business (B2B) auctions have become a dominant mechanism used by large shippers to procure contracts for transportation services from logistics companies. The bid analysis problem is of critical importance to shippers and determines which contracts are assigned to specific carriers and at what price. In practice this problem is further complicated by the consideration of shipper business rules, such as restrictions on carrier numbers, limits on the number of individual packages awarded and preferences for incumbent carriers. This paper examines the case in which bidding packages are mutually exclusive. This is referred to as a non-combinatorial auction. In practice, this type of auction is preferred to a full combinatorial auction because it allows the auctioneer (the shipper) to maintain control of the packages and creates much less cognitive strain for bidders (trucking companies). A mathematical programming model for the bid analysis problem is presented and heuristic construction algorithms and Lagrangian relaxation based algorithms are developed to solve the problem. Numerical results show that our Lagrangian relaxation based heuristics perform better than other heuristics and that the solutions are very close to optimal.

Introduction

Transportation service procurement is a critical task in the logistics operations of large shippers. Auctions have become a dominant price discover mechanism for this task. In this process, shippers intend to outsource their transportation functions to commercial carriers (trucking companies, for example) by letting them bid for periodically renewed contracts to serve specific origin destination pairs (lanes). While B2B auctions present shippers with opportunities to induce true prices from carriers, shippers are confused with such decision problems as how to determine the winning carriers and which bids to be assigned at what prices. The optimization problems generated by these auctions can involve thousands of lanes and hundreds of carriers. The sheer size of the problems faced by large shippers, as well as the fact that they have complicated business constraints to consider, make these problems very hard to solve.

Further, a procurement auction can be implemented with various auction mechanisms. In non-combinatorial auctions, shippers pre-specify bid packages before the auction; while in combinatorial auctions, carriers have the flexibility to define their own packages. The bid analysis problems for these two mechanisms are quite different.

In this paper, we will discuss how to model the bid analysis problem in transportation procurement auctions, particularly how to incorporate shippers' business requirements. This problem is modeled as a combinatorial optimization problem, further, greedy and optimization based heuristic algorithms are proposed to provide near-optimal solutions in reasonable time. Numerical experiments are also developed to examine the performance of our algorithms. In this paper, we focus on non-combinatorial procurement auctions which are still preferred by a majority of shippers in practice. In the end we will also discuss potential solution approaches to attack the bid analysis problem in combinatorial auctions, which is typically harder to solve.

In the following sections, first we briefly review the background for this research. Then we will discuss why non-combinatorial auctions are used by many shippers, and we will discuss typical shipper business requirements that arise in the bid analysis stage in transportation procurement auctions. Next, we propose a combinatorial optimization model for the bid analysis problem with incorporation of these business constraints. Because of the computational complexity of this problem, we develop greedy algorithms and a Lagrangian relaxation based algorithm for this problem. We then provide numerical results to analyze the experimental behavior of these algorithms. We end with some conclusions and discussion of extensions of this research.

Background

The procurement of freight transportation services is a critical component in large shippers' logistics operations. Shippers have long realized that depending solely on private fleets is inefficient and they are increasingly hiring commercial transportation companies under periodically renewed contracts. Over one-third of the $600 billion annual trucking business in the United States is fulfilled by for-hire common carriers (American Trucking Association, 2003). In practice, shippers typically select common carriers to fulfill their freight transportation demand based on competitive bidding in procurement auctions. This process, also called request for quotes (RFQ), allows shippers and carriers to develop strategic or tactic transportation solutions that benefit both parties.

To date, most of the procurement auctions in the transportation industry have been implemented as unit or non-combinatorial auctions in which carriers are allowed to bid only for individual packages that are pre-defined by shippers – these packages are mutually exclusive and each lane is included in only one package. While this type of auction is not as economically efficient as combinatorial auctions in which bidders have the freedom to build their own packages and make conditional bids, it has some nice properties and dominates the current transportation service procurement market. In practice, there are many potential advantages. First, the cognitive or computational strain placed on carriers and shippers is significantly reduced. Identifying efficient prices and developing good bids in complex auctions is no simple task. Second, it gives shippers more control over how lanes are grouped. Since only the shippers have reliable historical demand information, this may allow them to develop packages with less overall demand stochasticity than could the carriers. Such a reduction in stochasticity is beneficial to carriers who can rely on the income stream from such contracts as well as to shippers, who can count on reliable and timely service. Finally, carriers will often dedicate a sub-fleet to serve large shippers so they have no intention of leveraging existing contracts to make new ones more efficient. In this paper, we only consider the unit procurement auctions.

A transportation procurement auction involves three steps: bid preparation, bid execution and bid analysis/assignment. Caplice (1996) discussed the bid preparation stage where shippers determine how to combine lanes into packages. Gibson et al (1993, 1995) discussed the criteria to select candidate carriers as participants (pre-screening). The bid execution stage is concerned with participants' bidding strategies. Nisan (2000) and Abrache et al (2002) discussed various bidding languages designed to describe bidders' preference structures in combinatorial auctions. Song and Regan (2003) examined the bid construction problem from the carrier perspective in the context of combinatorial auctions and presents optimization based tools to construct bids. In this paper, we focus on the bid analysis stage after bids are submitted, that is, how shippers should analyze these bids and assign contracts to carriers in an optimal way. Note that the contracts have the following form. A shipper expects to have X loads on lane AB per week during the time of the contract. The carrier agrees to carry the loads at a pre-defined price, if the carrier has sufficient capacity when a request for service is made.

The shipper does not typically guarantee that it will have the loads nor does it specify how the loads will be distributed during the week. The carrier does not guarantee to carry the loads but guarantees a price, if it does carry the loads.

Bid analysis can be a daunting task for shippers even in unit auctions. The first issue is problem size – a transportation procurement auction can involve thousands of lanes and hundreds of carriers (Caplice and Sheffi, 2003, Elmaghraby and Keskinocak, 2003). If shippers can assign contracts solely based on bid price, the bid analysis problem would still be simple – a sorting algorithm can solve the problem very quickly since there is no interrelationship between different bid packages. What really makes it complicated is when sophisticated business rules are involved. For example, shippers may wish to select a limited number of carriers as their service providers due to the difficulty of managing too many accounts. And they may want to explicitly include carrier performance in the selection process, rather than viewing all pre-screened carriers as equal. As a result, shippers have to balance prices, costs associated with managing multiple accounts and expected service levels. These business constraints further complicate the bid analysis problem. For this reason, several third party logistics companies are dedicated to developing decision support tools for transportation procurement auctions, for instance, the Transportation Bid Collaboration tool developed by i2 inc. and OptiBid developed by Manhattan Associates.

These business constraints also vary with among shippers and industry applications. Caplice and Sheffi (2003) discussed the constraints found in transportation service procurement auctions and presented the general formulation for the bid analysis (carrier assignment) problem. These include:

- *Minimum / maximum number of winning carriers*: On the one hand, a shipper would not take the risk to put all their business into a single carrier's hand; on the other hand, they prefer to contract with a limited number of carriers both to reduce overhead costs associated with multiple suppliers and to give their core carriers more volume such that it can be a dominant customer for their core carriers.
- *Favor of incumbents*: It is typical for shippers to favor particular incumbents to be their core carriers at the lane, facility or system level; or wish to restrict some carriers from serving certain lanes. Caplice and Sheffi (2003) noticed "incumbents are often favored by 3% to 5% - especially on service-critical lanes to key customers or time-sensitive plants".
- *Back up concerns*: A shipper may require carriers to submit both bids as a primary and backup service provider.
- *Minimum / maximum coverage*: A shipper often wants to aggregate their demand and ensure the amount of traffic that a carrier wins within certain bounds, at a lane, facility or system level.
- *Threshold volumes*: Shippers can specify that if a carrier wins any freight (on a lane, from or to a facility, or system wide), it is of either a certain minimum threshold amount, or they win nothing at all.

- *Complete regional coverage*: Shippers may require every bid for services from a certain location or within a particular region to be able to cover all lanes from that location or region.
- *Performance factors*: For shippers, there are certainly tradeoffs between a carrier's bid prices and its level of services. A carrier may bid for less but does not have the capability to fulfill services they promised – this is called "lose the auction, win the freight" in practice.

Guo et al (2003) discusses how to incorporate some of these constraints into their carrier assignment models in unit procurement auctions for transportation services. Their formulation is somewhat different from ours. In our formulation, the items to be assigned are packages. In theirs, the items are lanes and the business constraint considered is shipper preference for specific carriers (expressed as penalty costs for carriers that are not preferred.) These penalties are modeled as negotiation costs at points of transit in their formulation. The bid analysis problems were solved using meta-heuristics and experimental results are presented in their paper.

The bid analysis problems in combinatorial auctions have also received wide attention in the research arena. This problem is coined the Winner Determination Problem in general combinatorial auctions. This problem, essentially a variant of the classic Set Partitioning Problem, has been studied by several groups of operations researchers and computer scientists (see for example de Vries and Vohra, 2001 and Sandholm and Suri, 2001) and has been applied in a variety of industries. Due to the complexity of this problem, shippers do not typically take business constraints into consideration in the bid analysis stage when implementing combinatorial auctions. For example, Ledyard et al (2002) reported on the execution of combinatorial auctions for Sears Logistics which solved winner determination problems without specific business constraints.

This paper examines the bid analysis problem with shipper's side constraints in unit or non-combinatorial procurement auctions. This research also provides insight on the general combinatorial auction problem and on similar problems in other industries.

A Bid Analysis Model with Shipper's Business Constraints

The fundamental problem at the bid analysis stage in transportation procurement auctions can be formulated as the following integer program:

$$\min \sum_{j \in J, k \in K} c_{kj} x_{kj}$$

$$s.t. \sum_{k \in K} x_{kj} = 1 \quad \forall j \in J \qquad (1)$$

$$\prod \qquad\qquad (2)$$

$$x_{kj} \in (0,1) \qquad (3)$$

(BAP)

where:

j : a bid package in set J which may or may not include multiple lanes
k : a bidding carrier in set K ;
c_{kj} : the shipper's cost to select carrier k to serve package j;
x_{kj} : is a binary variable indicating whether a carrier k wins a package j ;
$\prod$: any business or logical constraints;

  The objective function of the bid analysis problem BAP is to minimize shipper's total costs to procure transportation services for a group of lanes in set L. Note that the cost function can be defined to incorporate non-price parameters such as service performance ratings in addition to prices. The first constraint ensures that each package is assigned to one and only one carrier. The second constraint models specific business constraints defined by shippers.

  Note that packages are mutually exclusive in a unit or non-combinatorial auction, that is, $j_1 \cap j_2 = \varnothing$. As a result, no lane will appear in more than one package. Further note that without the second constraint set, the bid analysis problem can be easily solved by sorting the bid price for each package and assigning a package to the bidder with the least price. However, when business constraints are incorporated, it becomes a very hard problem.

  The complete incorporation of all possible business constraints requires building a sophisticated decision support system and is beyond the scope of our paper. In the following, we focus on those constraints discussed in Caplice and Sheffi (2003). In particular, we clearly modeled these business requirements as side constraints in our model: *maximum / minimum number of winning carriers, incumbent preference, maximum / minimum coverage, performance factors*. The service backup issue can be illustrated in the bid preparation stage by requiring each carrier to submit both primary and alternate bids and hence is not considered here. The complete regional coverage constraint can be addressed by combining all traffic lanes from that location or within that region into a single bid package at the bid preparation stage. For performance factors, some shippers conduct pre-screening activities on bidder's qualifications at the bid preparation stage to ensure minimum level of services (Ledyard et al, 2002); another way to model this constraint is to use an adjusted price instead of pure bid price for the cost function. Essentially, this allows the shipper to penalize carriers that have not been pre-screened without completely eliminating them from consideration. Finally, we assume that the freight volume on each lane is not separable.

  Now the bid analysis problem with shipper's business constraints and penalty costs in unit auctions can be written as follows:

$$\min \sum_k \sum_j c_{kj} x_{kj} + \sum_k p_k y_k$$

$s.t.$

$$\sum_k x_{kj} = 1, \qquad \forall j \in J \qquad (4)$$

$$K_{min} \le \sum_k y_k \le K_{max}, \qquad (5) \qquad \textbf{(BAP-P)}$$

$$T_{min}^k y_k \le \sum_j x_{kj} \le T_{max}^k y_k, \quad \forall k \in K \qquad (6)$$

$$y_k, \ x_{kj} \in (0,1) \qquad (7)$$

where:

p_k is the penalty cost for carrier k to be selected as a winner, $p_k \ge 0$;

K_{max} is the maximum number of carriers to be selected as winners;

K_{min} is the minimum number of carriers to be selected as winners;

T_{max}^k is the maximum number of packages (lanes) assigned to carrier k if it wins;

T_{min}^k is the minimum number of packages (lanes) assigned to carrier k if it wins;

In this model we also have $T_{max}^k \ge T_{min}^k \ge 1$ and $K_{max} > K_{min} \ge 1$. In addition to x_{kj}, we have another decision variable y_k – a binary variable indicating whether a carrier is a winner or not;

The objective function of the BAP-P problem minimizes total procurement costs including the bid prices and the penalty costs to manage multiple carrier accounts. As shown in Figure 1, there is actually a trade-off between these two costs: a very large carrier base will reduce bid prices, i.e., the actual transportation costs; however, contracting with too many carriers will increase shipper's overhead costs.

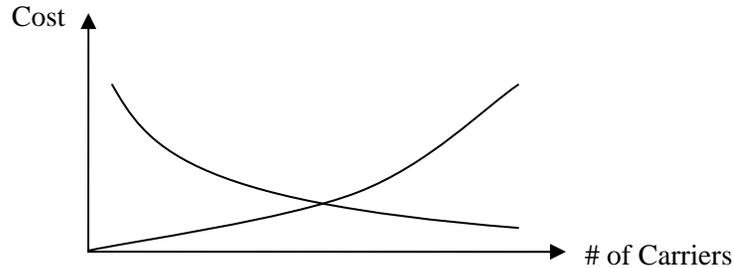Figure 1. Relationship between procurement costs and number of winners

Further, note that a penalty cost can also be used to capture the shipper's favoring of specific carriers at the system level – incumbents have a zero penalty cost and non-incumbents have a positive penalty cost. The first constraint in the BAP-P formulation ensures that each package (lane) is served by one and only one carrier. The second

constraint restricts the number of winners (size of carrier base) across the system in the final assignment. Constraint set (6) indicates the minimum and maximum coverage for each winner. That is, shippers want to make sure that a carrier carries a minimum and/or maximum amount of traffic volumes if this carrier is selected as a winner ($y_k = 1$). Though we only model this constraint at the system level, it can be easily modified to express restrictions at facility level. Also note that $T_{\max}^k$ can be used to model a carrier's capacity. For example, a small carrier may bid for more than it can handle.

Constraint (6) is also a coupling constraint which models the following relationship between decision variables x_{kj} and y_k:

$$y_k = \begin{cases} 1, & \text{if and only if } \sum_j x_{kj} \geq 1 \quad (8) \\ 0, & \text{otherwise} \quad\quad\quad\quad (9) \end{cases}$$

Next we analyze the complexity of this problem by transforming it to a Capacitated Fixed Charge Facility Location Problem (CFCFLP). The CFCFLP problem finds the number and location of facilities to serve a set of demand nodes while minimizing the sum of fixed facility location costs and the transportation costs between facilities and demand nodes (see for example Daskin, 1995).

First note that if we add another coupling constraint to the BAP-P problem:

$$x_{kj} \leq y_k \quad \forall k \in K, j \in J \quad\quad (10)$$

it will not change the problem structure and we still have the same problem. Now by removing constraint (5) on the number of winners, the problem turns into an instance of the CFCFLP problem with non-fractional demand. In our problem, a demand node is a bid package with unit demand, a candidate facility is a carrier $k \in K$, the transportation cost between a facility and demand node is the carrier's bid price for that package, and the facility cost corresponds to that carrier's penalty cost. Finally, the cost per unit distance per unit demand is 1. Since the CFCFLP problem is known to be NP-hard, by adding constraint (5) and non-fractional demand constraint, the BAP-P problem is also NP-hard.

Proposition: The bid analysis problem BAP-P with shipper's business constraints in unit procurement auctions is an NP-hard problem.

Inspired by the resemblance of the BAP-P problem to the facility location problem, we developed the following greedy and optimization-based heuristics to solve this bid analysis problem.

Greedy Algorithms

These algorithms either construct a solution from the ground up or try to improve from an initial solution. In addition we combine the two approaches in a hybrid heuristic. They are "greedy" in nature because in each step we choose the best carrier or bid package that can reduce total costs as much as possible.

Heuristic Construction Algorithms

We use two approaches to construct a base of winning carriers: sequentially adding more carriers into or dropping carriers from that base. We call the first one a Modified ADD algorithm (MADD) and the second one a Modified DROP algorithm (MDROP).

In the MADD algorithm, we gradually add more carriers into the winner set to see if we can further improve the solution. At the beginning, we assume each bid package is assigned to a dummy carrier with very large bid prices. Then at each iteration, we select a winner who can reduce the total cost at the greatest amount or increase the total cost at the least amount without violating other constraints. This procedure is continued until either, (1) the minimum-number-of-winners constraint is satisfied and adding more carriers will result in cost increment; or, (2) the maximum-number-of-winners constraint will be violated if more carriers are added. Letting TC = total cost including transportation costs and penalty costs, the MADD algorithm is outlined in Appendix 1.

Specifically, we select winners with iterative steps: let the set of winning carriers be K_n at iteration n. First we do not consider the $[T_{\min}^k, T_{\max}^k]$ bound and for each carrier $r \notin K_n$, we compute TC_r – the total cost if this carrier r is added into K_n. Then we temporarily add the carrier with the minimal TC_r to the winner set and assign this carrier with all packages which it has a less bid price.

In this procedure, some winners might violate the $[T_{\min}^k, T_{\max}^k]$ bound, so we need to balance traffic lanes among winners in next step. If a carrier k wins only a number of packages less than $T_{\min}^k$, then we balance the traffic volume in the following way. For each package carrier k does not win, calculate the incremental of bid price if this bid package is assigned to k, assign these packages to this carrier according to the increasing order of this bid price increment under the condition that other carriers still have enough packages. This process is continued until constraint (6) is satisfied for each carrier. A similar balancing process can be implemented for those winners with the number of assigned packages greater than $T_{\max}^k$.

Note that after we balance traffic lanes among winners, the total cost could possibly increase could exceed the cost of adding another carrier. If that occurs, repeat the process to check whether adding another carrier instead will result in a better

balanced assignment. As a result, there might be a back-and-forth process between the third and fourth steps in the procedure.

The MDROP algorithm works in a similar manner. Initially for each bid package, we select the carrier with the minimum bid price to serve that package and add that carrier into our set of winners. If the total number of winning carriers exceeds K_{min}, then we check which carrier to be dropped will result in the maximum savings. We greedily continue our search until either no further cost reductions can be found or the total number of winning carrier drops to K_{min}. The lane balancing step is similar to that in the MADD algorithm.

The procedure of an MDROP algorithm is outlined in Appendix 2.

In the MADD and MDROP algorithms, we add winners first and balance lanes second. It is also noticed that this process can be reversed, that is, we can balance lanes first and add winners second. The procedure is similar so we omit the details here.

Heuristic Improvement Algorithms

Given a feasible solution to the bid analysis problem using either construction algorithm, we can further improve on the solution through exchange of bid packages or substitution of carriers. In particular, a heuristic improvement algorithm can be implemented following these two steps:

1. Keep the winning carrier set, exchange bid packages among carriers within this set. This reduces to an assignment problem where bid packages are assigned to a fixed number of carriers with minimal total bid prices. Heuristics for assignment problems can be applied here with small modifications.

2. Keep the number of winning carriers and assignment of bid packages, but substitute one winner with another carrier not in the set of winners to see whether solutions can be further improved. This approach is easy to implement.

Finally, a combination of heuristic construction algorithm and improvement algorithm will result in a hybrid heuristic algorithm. In this paper, we are more interested in optimization based heuristic algorithms than greedy algorithms. Indeed, we found a Lagrangian relaxation based approach performs much better with reasonable computing time.

Lagrangian Relaxation based Approach

In this section, we propose a Lagrangian relaxation based approach to solve the bid analysis problem BAP-P. Lagrangian relaxation is a very efficient optimization-based approach to solving a number of combinatorial optimization problems (Fisher, 1981). The general idea is first to relax some side constraint of the original problem and to produce a Lagrangian problem that is easy to solve and whose optimal solution provides a lower bound for the original problem; a feasible solution is further constructed

for the original problem from this optimal solution with some heuristic and provides an upper bound; this procedure is repeated to reduce the gap between the lower and upper bound by changing Lagrangian multipliers.

The structure of the bid analysis problem suggests a number of relaxations on different constraints. Due to the strong similarity between this problem and the facility location problem, we only dualize constraint (4) in the BAP-P problem with unsigned Lagrangian multipliers $u = (u_1...u_j...)$ and obtain the following Lagrangian relaxation problem:

$$\max_{u} \ \min_{x,y} \ \sum_{k}\sum_{j}(c_{kj}+u_j)x_{kj} + \sum_{k}p_k y_k - \sum_{j}u_j$$

$s.t.$

$$K_{\min} \le \sum_{k}y_k \le K_{\max} \qquad\qquad (11) \qquad \textbf{(BAP-P-LR)}$$

$$T_{\min}^k y_k \le \sum_{i}x_{ki} \le T_{\max}^k y_k, \quad \forall k \in K \qquad (12)$$

$$y_k, \ x_{kj} \in (0,1) \qquad\qquad (13)$$

Next we discuss how we solve each instance of this relaxed problem BAP-P-LR to optimality in polynomial time given a vector of Lagrangian multipliers.

First note that the relaxed Lagrangian problem BAP-P-LR can be modeled as a network flow problem. In the following graph, we need to push a flow with a total volume L from dummy node s to dummy node t via intermediate node k (carrier) and j (package) at the minimal costs. Each node k has a capacity bound $[T_{\min}^k, T_{\max}^k]$ and a penalty cost p_k, each edge linking k and j has an adjusted cost $\tilde{c}_{kj} = c_{kj} + u_j$.
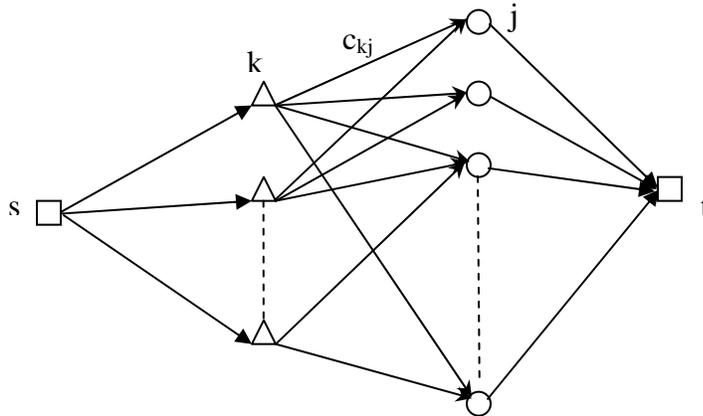
Inspired by this observation, we developed the following solution approach. For each carrier, we first build a list of bid packages T_k. We associate each carrier k with a sorted list of $\hat{c}_{kj}$, then we continuously add a package j into T_k from an increasing order of $\hat{c}_{kj}$. This procedure stops when either (1) the size of T_k is equal to T^k_{min} and the next $\hat{c}_{kj}$ in the list is greater than zero; or (2) $\hat{c}_{kj}$ is still less than zero but the size of $T_k = \sum_j x_{kj} = T^k_{max}$.

Now for each carrier, we have a list of candidate packages T_k and the total cost $TC_k = p_k + \sum_j \{\hat{c}_{kj} \mid j \in T_k\}$. Next we sort all carriers in increasing order of TC_k. Then we add the K_{min} number of carriers with smallest TC_k into the winner set K_{opt}; for the rest of carriers, we continue to add those with $TC_k < 0$ into the winner set K_{opt} until the constraint $\sum_k y_k \leq K_{max}$ is violated.

Finally, we let $y_k = 1$ for all $k \in K_{opt}$ and $y_k = 0$ for other carriers. Further, we set $x_{kj} = 1$ for all bid packages in the list T_k, that is, $k \in K_{opt}$ & $j \in T_k$, and $x_{kj} = 0$ for others.

Now this solution is an optimal one to the Lagrangian problem BAP-P-LR with u_j and is also a lower bound to the original bid analysis problem. In addition, this solution approach can be implemented in polynomial time. In each iteration, the time to solve a relaxed Lagrangian problem is $O(sK * Sort(sJ))$, where sK is the total number of carriers and $Sort(sJ)$ is the time to sort bid prices for sJ number of bid packages. There are many good sorting algorithms with polynomial running time.

Once we can find an optimal solution for the Lagrangian problem, we need to construct a feasible solution for the original BAP-P problem.

Note that an optimal solution for the Lagrangian problem may violate constraint 4 ($\sum_k x_{kj} = 1$) in the BAP-P problem with either of the following two variable sets:

1. A bid package is not covered, that is, $\sum_k x_{kj} = 0$ for some j;

2. A bid package is covered by more than one carrier, that is, $\sum_k x_{kj} \geq 2$ for some j;

For the first case, we simply assign such a bid package j to the best carrier k^* such that: $k^* = k^* \mid c_{k^*i} \leq c_{ki}, \forall k \in K_{opt}$, where K_{opt} is the optimal winner set.

For the second case, we will risk making some carriers win less packages than their $T_{\min}^k$ if we simply remove redundant carriers for each bid package. As a result, the following heuristics is developed to tackle with this case.

(1) if $\sum\limits_{k \in K_{opt}} T_{\min}^k > sJ$, where sJ is the total number of bid packages;

This situation often occurs when shippers have to choose some large carriers but not all of them. However, the optimal solution might pick more carriers than shippers can afford. As a result, we need to either remove some carriers from the set of winners or substitute some carriers with others having less $T_{\min}^k$. Let $f = \sum\limits_{k \in OPT} T_{\min}^k - sJ$, the procedure can be implemented as:

▪ If $\sum\limits_{k \in K_{opt}} y_k > K_{\min}$, for each carrier k, compute the incremental cost of removing this carrier and assigning its packages to other carriers in K_{opt}. Remove the carrier who will result in the minimal increment of costs until $\sum\limits_{k \in K_{opt}} T_{\min}^k \leq sJ$ is satisfied.

▪ If $\sum\limits_{k \in K_{opt}} y_k = K_{\min}$, for each carrier $k \in K_{opt}$, compute the incremental cost of by removing this carrier from the set of winners and assigning its lanes to other carriers not in K_{opt}. Substitute the carrier whose removal will result in the minimal increase in costs with its corresponding carriers not in K_{opt} until $\sum\limits_{k \in K_{opt}} T_{\min}^k \leq sJ$ is satisfied.

(2) if $\sum\limits_{k \in K_{opt}} T_{\min}^k \leq sJ$

In this situation, we only need to reassign packages among winning carriers such that each of bid packages is served by only one carrier.

▪ For each package $j \mid \sum\limits_k x_{kj} \geq 2$, remove redundant carriers as follows:

Set $x_{k^*j} = 1$, if $k^* = \min\limits_k \{c_{kj} \mid x_{kj} = 1\}$; $x_{kj} = 0 \quad \forall k \neq k^*$

▪ Now each package j is connected to only one carrier, then we check whether each carrier's $T_{\min}^k$ constraint is satisfied. Split the set of winners K_{opt} into two subsets:

$P = \{k \mid \sum\limits_j x_{kj} < T_{\min}^k\}$ and $Q = \{k \mid \sum\limits_j x_{kj} \geq T_{\min}^k\}$

For each $k \in Q$, sort $c_{kj} \mid x_{kj} = 1$ into a list with increasing order, identify $T_{\min}^k$ number of packages at the top of this list, put the rest of packages into a set RQ_k,

note the size of $RQ_k = \sum_j x_{kj} - T_{\min}^k$. Let $RQ = \underset{k \in Q}{\cup} RQ_k$. This set includes all candidate packages that can be reassigned to carriers in P.

Now for each package $j \in RQ$, compute the incremental price if it is not served by its assigned carrier $k' \in Q$, instead, by a carrier $k \in P$, that is, $c_{kj} - c_{k'j}$. Start from the triplet (k, k', j) with the least incremental increase in price, let $x_{kj} = 1$ and $x_{k'j} = 0$, remove package j from set RQ. Once carrier k has enough demand such that $\sum_j x_{kj} = T_{\min}^k$, remove k from set P. Repeat this procedure until set P is empty.

Now this solution is indeed a feasible one to the original bid analysis problem BAP-P, and it also provides an upper bound to the problem. In addition, this heuristic algorithm of finding feasible solution can be implemented in $O(sJ * sort(sK))$ time.

We can further improve the Lagrangian lower bound and reduce the gap between the upper bound and lower bound. There are alternative ways to do this, among them is the well-known subgradient search method. Let $Z_0(u_i^n)$ be the optimal solution from the Lagrangian problem BAP-P-LR (lower bound) and x^n, y^n be the optimal assignment at iteration step n, and let Z^* be the feasible solution (upper bound), the subgradient search method starts with an initial value u^0 for the Lagrangian multipliers and updates them over the iterations as:

$$u_j^{n+1} = u_j^n + t_n(\sum_k x_{kj} - 1)$$

where:

$$t_n = \frac{\lambda_k(Z^* - Z_0(u_j^n))}{\sum_j (\sum_k x_{kj}^n - 1)^2}$$

In the above equation, t_n is a scalar satisfying $0 < t_n \le 2$, normally we have $t_0 = 2$ and it will be divided in two whenever $Z_0(u_i^n)$ has failed to increase in a fixed number of iterations.

To summarize, the procedure for Lagrangian relaxation based approach is as the following:

1. Relax constraint (4), start from $u = u_0$, solve a relaxed Lagrangian problem BAP-P-LR to optimality;
2. Find a feasible solution for the original BAP-P problem from the optimal solution of BAP-P-LR using the heuristics we describe;

3. Check whether any stoping rule is satisfied, if not, go to the next step, else stop the program. Common stop rules include whether the lower bound is close to the upper bound and whether there have been too many iterations;
4. Update Lagrangian multipliers u using the Subgradient method and return to step 1.

Experimental Results

Numerical experiments were developed to examine the performance of our heuristics including MADD, MDROP and Lagrangian relaxation based method. In particular, we implemented these algorithms on a suite of randomly generated problems and compared their solution qualities and running time.

In order to implement our Lagrangian relaxation based method, we need to specify several system settings. First, as indicated above, the running time of the Lagrangian relaxation based method is closely related to the performance of the sorting algorithm. In our experiments, we used the quicksort algorithm (Cormen et al, 2001) with a running time $O(n \log n)$. As a result, the running time of our Lagrangian relaxation based method is $O(K * L * \log L)$.

The solution quality of Lagrangian relaxation based method also heavily depends on the choice of initial values for Lagrangian multipliers. We explored a few initial values and found the following two perform best on average:

$$u_j^0 = u^0 = \sum_{k,j} c_{kj} / sJ + \sum_k p_k \text{ and } u_j^0 = \sum_k c_{kj}$$

As a result, we use these two to generate initial values for Lagrangian multipliers in parallel and stop the program whenever either of them finds a near optimal solution. In addition, the subgradient method is used to update Lagrangian multipliers during the program. The initial value of positive scalar λ_k is set to 2, and is halved whenever the optimal solution for the relaxed problem cannot be improved in 4 successive iterations.

Further, the following rules are deployed to determine whether we should stop the iterations of Lagrangian relaxation based method:

1. Optimal solution is found (optimal solutions for Lagrangian problem are also feasible to the original problem, or the best upper bound is equal to the best lower bound);
2. Near optimal solution is found (upper bound – lower bound < 0.001);
3. The total number of iterations exceeds 2000 (we allow the program to run up to 4000 iterations if the solution is not good and the running time is small);
4. λ_k is too small ($\lambda_k < 1e-10$);

In this experiment, we use solution quality and running time to measure the performance of different algorithms. In terms of solution quality, we examine the gap between solutions from our heuristic algorithms and optimal solutions from commercial optimization software CPLEX version 8.1. For very large problems which CPLEX cannot solve to optimality within a working day, we evaluate the performance based on the gap between upper bound and lower bound in Lagrangian relaxation based method, and the gap between greedy algorithm solutions and Lagrangian upper bound.

Two data sets are developed for this purpose in our experiments. In practice a transportation procurement auction involves a dozen to several hundreds of carriers and a few hundreds to ten thousands of lanes (Caplice and Sheffi, 2003). Therefore we designed our test data sets including a set of small problems (20 to 50 carriers and 200 to 400 lanes) and another set of large problems (100 to 500 carriers and 2000 to 10000 lanes). It is noted in our experiments that CPLEX can solve most problems in the first set within a working day, but it cannot guarantee to solve the large problems in the second data set even if given much longer computation time. (All experiments conducted on an AMD Athlon 1200 machine with 512 MB memory). The size of each problem set is listed in Table 1 and 2. For each type of problem, we tested a dozen instances and the results are presented as the average over those instances.

Input data for each problem includes each carrier's bid prices, penalty cost, minimum and maximum number of lanes if this carriers is a winner, minimum and maximum number of winners. In our experiments, a carrier's bid price c_{kj} is randomly distributed between 10 and 100, and the penalty cost is randomly distributed between 0 and 3% of total bid prices. Please note that this method of generating test data is without loss of generality because of the structure of the unit auction. If this were a general combinatorial auction then input data would have to come either from a real world dataset or from data generated over a transportation network. We set $K_{\min} = 5$ and $K_{\max}$ is set to be the number of bidders. In addition, each carrier has a $T_{\min}^k$ that is uniformly distributed over $\left[1, \, sJ/1.5K_{\max}\right]$ and $T_{\max}^k \in \left[sJ/1.5K_{\min}, \, sJ\right]$.

The numerical results are summarized reported from Table 1 to Table 4. Table 1 lists both optimal solution by CPLEX and near-optimal solution obtained using the heuristic algorithms for small problems. The gap between the lower bound and upper bound solution given by the Lagrangian based method is very tight and the ratio between them is above 97% almost in all cases; in addition, the Lagrangian feasible solution is also very close to the optimal solution. Surprisingly, even though greedy algorithms do not perform as well as the Lagrangian based method, their solutions are close to optimal as well. Further, the solution by MDROP algorithm is slightly better than the MADD solution, but the difference might not be statistically significant.

As shown in Table 2, the computation time used by CPLEX is not comparable with the heuristic algorithms. The CPLEX solution time increases exponentially with the size of problems and in some cases this time exceeds 10 hours for a relatively small problem while the heuristic algorithms can solve these in less than 1 minute. As was

expected, the time used by the Lagrangian based method is slightly higher than those of the greedy algorithms.

Table 1 Average Solution Quality of Small Bid Analysis Problems Under Alternative Heuristics

Case Index	1	2	3	4	5	6	7	8	9
# of carriers	20	20	20	30	30	40	40	40	50
# of lanes	200	300	400	300	400	300	400	500	400
Lower / Upper	99.8%	99.9%	99.3%	99.6%	96.9%	97.4%	97.9%	97.5%	97.9%
Upper / CPLEX	1.0	1.0	1.0	1.0	1.0	1.001	1.001	1.0	1.0
MADD / CPLEX	1.01	1.0	1.001	1.007	1.003	1.009	1.004	1.002	1.003
MDROP / CPLEX	1.0	1.0	1.001	1.0	1.0	1.003	1.001	1.001	1.001

Table 2 Average Computation Time for Small Bid Analysis Problems (Minutes)

Case Index	1	2	3	4	5	6	7	8	9
CPLEX	0.5	2.2	9.2	10.8	66.3	66.2	137.5	231.0	192.5
Lagrangian	0.6	0.3	0.6	0.5	0.7	0.6	0.8	0.7	0.7
MADD	0.03	0.03	0.03	0.04	0.04	0.05	0.06	0.06	0.07
MDROP	0.00	0.01	0.01	0.02	0.03	0.03	0.04	0.04	0.05

The performance of the Lagrangian based method is constant with the increase of problem size as indicated in Table 3 and 4. Even with a very large problem size of 500 carriers and 10,000 lanes, the gap between lower bound and upper bound is less than 1%. And its computation time is less than 4 hours.

However, the performance of the greedy algorithms deteriorates when the problem size is relatively large. Even though the average ratio between their solutions and feasible solutions given by Lagrangian based method (upper bound) is less than 1.1 on average, we have spotted cases where this ratio exceeds 1.3. The advantage of these greedy algorithms are clearly fast computational time.

Table 3 Average Solution Quality of Large Bid Analysis Problems Under Alternative Heuristics

Case Index	11	12	13	14	15	16	17	18	19
# of carriers	100	100	200	200	300	300	400	400	500
# of lanes	2000	4000	4000	6000	6000	8000	8000	10000	10000
Lower/Upper	99.2%	96.9%	97.9%	99.0%	99.6%	99.3%	99.0%	99.1%	99.0%
MADD/Upper	1.057	1.051	1.063	1.063	1.070	1.067	1.068	1.090	1.080
MDROP/Upper	1.056	1.050	1.058	1.062	1.065	1.066	1.067	1.076	1.071

Table 4 Average Computation Time for Large Bid Analysis Problems (Minutes)

Case Index	11	12	13	14	15	16	17	18	19
Lagrangian	6	14	31	48	76	101	136	181	225
MADD	0.4	0.4	0.6	1	1.1	1.4	2.1	4	7.6
MDROP	0.5	1.1	3.9	6.6	13.9	20	34	46	69

In summary, both greedy algorithms and Lagrangian based heuristics have an unbeatable advantage over exact algorithms. The latter cannot be guaranteed to solve practical bid analysis problems. Further, the Lagrangian based algorithm can provide feasible solutions that are very close to optimal.

Conclusion and Extensions

Procurement auctions have been used by shippers to contract with common carriers for several years. E-commerce further boosted this price discovery mechanism. Shippers used to select carriers based solely on bid price, however, this may lead to sub optimal choices if non-price attributes are not considered. In addition, shippers have other sophisticated business considerations such as "core carrier programs" in which shippers want to give more transportation volume to fewer carriers.

In this paper, we considered the bid analysis problem with shipper's business requirements in the popular unit or non-combinatorial auctions for the procurement of transportation services. That is, how shippers should select winning carriers and assign bid packages among them while taking their business rules explicitly into consideration. Further, a combinatorial optimization model was proposed to incorporate such business considerations as limitations on the number of winners and winning volume, incumbent preferences etc. While the problem is NP-hard, greedy and optimization based heuristic algorithms were developed to solve this problem. In addition, numerical experiments were designed to measure the performance of different algorithms. The results showed that heuristic algorithms are much faster than the exact algorithms included commercial software such as CPLEX. We further showed that greedy algorithms can provide very good solutions for small problems but that solution quality deteriorates for large problems. However, the Lagrangian based method is consistent in terms of solution quality and can fairly quickly generate solutions very close to optimal regardless of problem size.

While this work represents an effort to model and solve the sophisticated bid analysis problems in transportation procurement auctions, several topics need to be examined and elaborated in the future. First, while this work modeled the most common business considerations explicitly, these rules may vary in practice from shipper to shipper. In addition, shippers may prefer to conduct sensitivity analysis to determine which business constraints should be included in the model.

In this paper, the bid analysis problem was considered in the context of unit auctions. While this will likely be the primary procurement method for quite some time, it has been observed that combinatorial auctions are more economically efficient and

better characterize the economies of scope properties inherent in transportation services. Shippers, especially large ones, are increasingly designing combinatorial auctions to procure transportation services. The bid analysis problem is much more complicated in those auctions and can be formulated as follows:

$$\min \sum_k \sum_j c_{kj} x_{kj} + \sum_k p_k y_k$$

$$s.t.$$

$$\sum_k \sum_j a_{ij} x_{kj} = 1, \ \forall i \tag{14}$$

$$K_{\min} \leq \sum_k y_k \leq K_{\max} \tag{15}$$

$$T_{\min}^k y_k \leq \sum_i \sum_j a_{ij} x_{kj} \leq T_{\max}^k y_k, \quad \forall k \tag{16}$$

$$y_k, \ x_{kj} \in (0,1) \tag{17}$$

where $i \in L$ is the index of lanes, and a_{ij} is a binary coefficient indicating whether lane i is included in bid package j.

Note that even without any business constraints, this Winner Determination problem is very hard and can be reduced to a Set Partitioning problem. Most of the past research on winner determination problems in combinatorial auctions has focused on the pure set partitioning problem. However, methodologies for incorporating non-price business factors have not yet emerged in the literature. That issue is the topic of our ongoing research.

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Appendix 1

```
                              ┌──────────────────┐
                              │ 1. Find one carrier
                              │ with least TC for
                              │ all lanes
                              └──────────────────┘
┌──────────────────┐                  │
│ 7. Permanently   │                  │
│ add this carrier │─────────────────→│
│ into carrier base│                  ▼
└──────────────────┘          ╱               ╲      No
                             ╱  2. Carrier size is ╲──────────→
                             ╲  less than K_max?   ╱
                              ╲               ╱
                                   │ Yes
                                   ▼
                            ┌──────────────────┐
                            │ 3. Search a carrier s.t.
                            │ TC is reduced the most
                            └──────────────────┘
                                   │         ▲
                                   ▼         │
                            ┌──────────────────┐
                            │ 4. Balance traffic lanes
                            └──────────────────┘
                                   │
              Yes                  ▼
         ←──────────    ╱  5. is TC reduced?  ╲
                         ╲                    ╱
                                   │ No
                                   ▼
              Yes          ╱ 6. Carrier size is ╲
         ←──────────        ╲ less than K_min?  ╱
                                   │ No
                                   ▼
                                ( Stop )
```

2. Carrier size is less than K_{max}?

6. Carrier size is less than K_{min}?
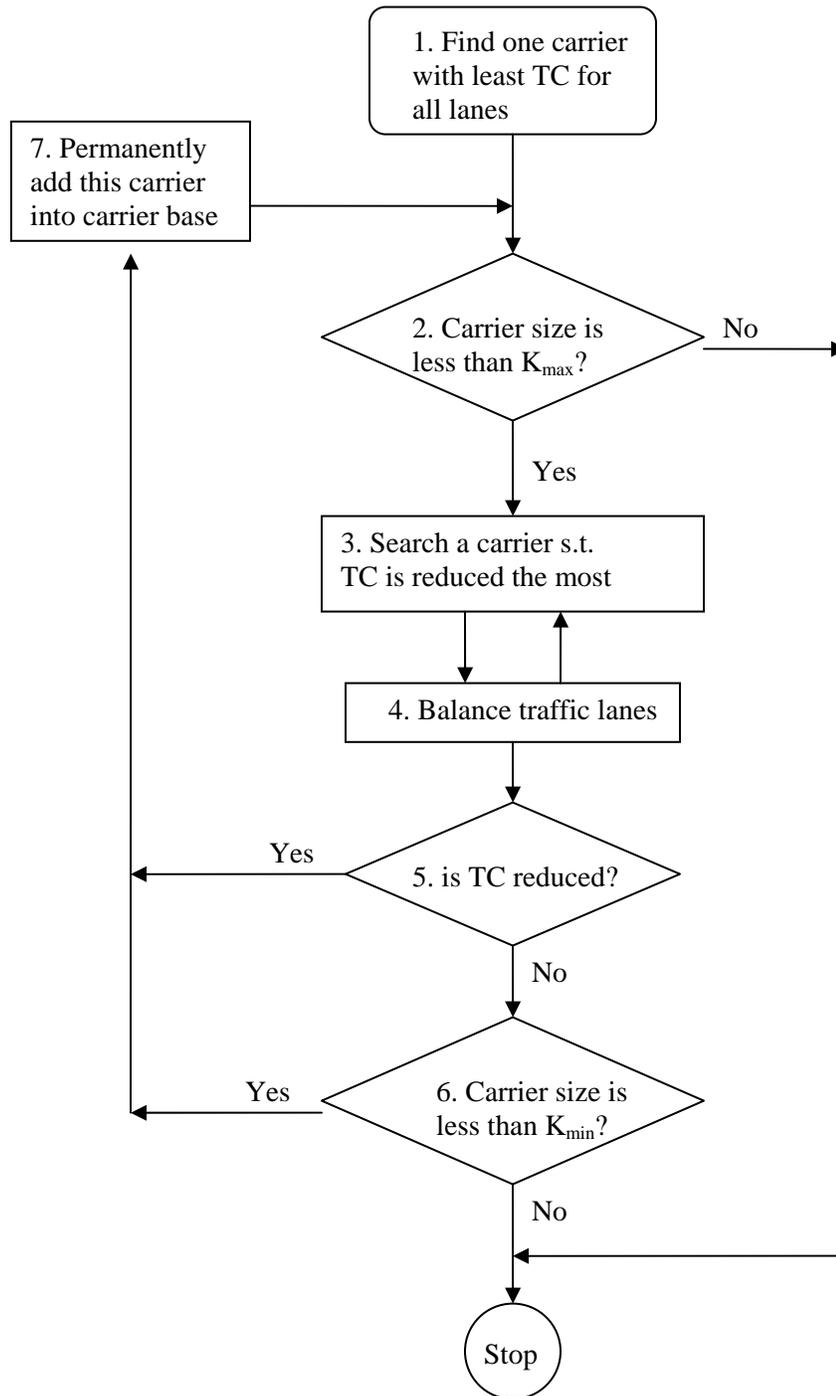
Figure 2 Flowchart of MADD Algorithm for the Bid Analysis Problem

Appendix 2

```
                            ┌─────────────────────┐
                            │ 1. Assign the least-│
                            │ bid-price carrier to│
                            │ each lane           │
                            └─────────────────────┘
┌─────────────────┐
│ 6. Permanently  │
│ drop this carrier│
│ from carrier base│
└─────────────────┘

        2. Carrier size is          No        MADD
        greater than K_min?

              Yes

        3. Search for a carrier s.t.
        TC is reduced if it is removed;
        Balance traffic volume

   Yes
        4. is TC reduced?

              No

   Yes
        5. Carrier size is
        greater than K_max?

              No

             Stop
```
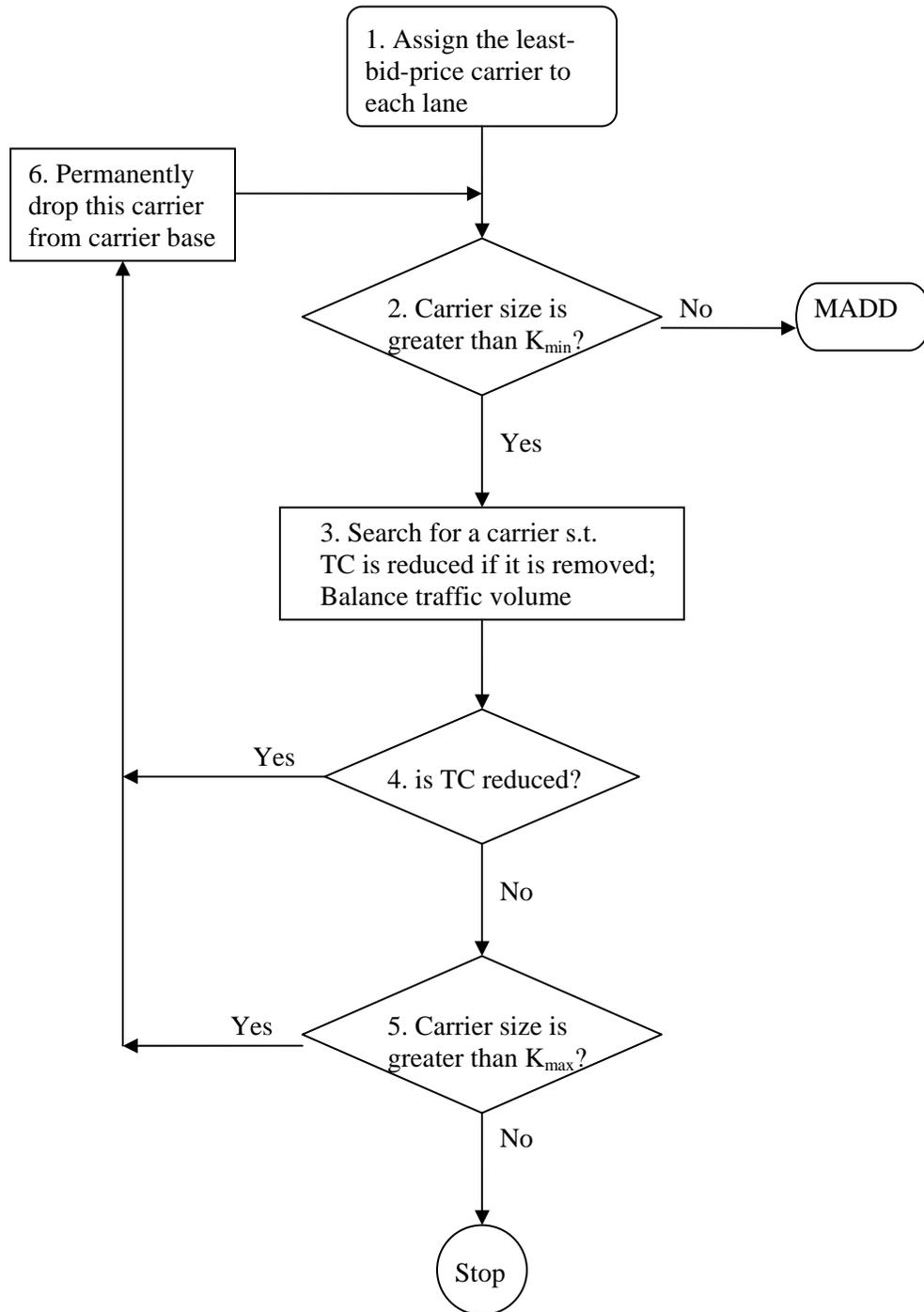
Figure 3 Flowchart of MDROP Algorithm for the Bid Analysis Problem